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Direttore: Prof. Massimo Onofri

CONDITIONAL SENTENCES
Truth Conditions And Probability

Tutor:
Prof. Alberto Mura
Prof.ssa Dorothy Edgington

Dottoranda:
Elena Nulvesu

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INTRODUCTION

Conditional statements have been subjects of several discussions since ancient age. Indeed linguistic constructions like "If *p,* (then) *q*" have always interested many philosophers and logicians because of their central role in common reasoning: every day we think and act in accordance with conditional statements. Unfortunately, the use of these sentences in theoretical and practical reasoning is problematic, leading often to absurdities:

[1] "If Berlusconi dies, Prodi will win the Elections. If Prodi wins the Elections, Berlusconi will resign immediately after the Elections. Therefore if Berlusconi dies, Berlusconi will resign immediately after the Elections."

[2] "I think Tom must be at home because the lights are on. And if he were not at home the lights would be off."

[1] is a classical transitive schema whose conclusion is clearly absurd although it is a valid representation in deductive theoretical reasoning. [2] is a typical non-inclusive theoretical reasoning that might be easily invalidate by the additional information that sometimes Tom forgets to switch off the lights.

Not less problematic is the use of conditionals in practical reasoning:

[3] "I have heart disease. To decrease the odds of a heart attack I should take medicines."

Looking at example [3] from another point of view it seems that people taking those medicines could have a heart attack easier than others. Misunderstanding like this could mislead the decision maker! So it is very important to pay attention to the action every conditional is affecting.

So, in front of the problematic but essential role of statements like "If *p,* (then) *q*" it is not exhaustive to identify a conditional simply with a sentence characterized by a link between the antecedent (*p)* and the consequent (*q*). A theory of conditionals must be able to show their great importance, when they are acceptable and when

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they are truth or simply assertive. For certain, this is not an easy task and, although a lot of progress has been made in this field, a genuinely unified theory of conditionals does not exist yet. Indeed, some theses seem good only for indicative and not for counterfactual conditionals (or vice versa) while others work well with simple but not with compound ones. A so-called unified theory has to be applied to all of these different accounts of conditionals.

Just to be clear, a simple conditional is that one where the conditional connective occurs once (connecting antecedent and consequent), and a compound conditional is a compound sentence containing occurrences of conditional connectives in some of its proper sub-sentences.

Regarding the difference between indicative and counterfactual conditionals, I think there is no better way than the following example to understand it:

[4] "If Oswald didn't shoot Kennedy, someone else did." *(Non-counterfactual or indicative conditional)*

[5] "If Oswald hadn't shot Kennedy, someone else would have." *(Counterfactual or subjunctive conditional)*

This is a paradigmatic illustration because it allows to say the first proposition is definitely unquestioned and the second is denied.[1] Indeed, unless we are not any theorist of conspiracy, we can reject [5] in spite of accepting [4]. Moreover, another dissimilarity--despite not crucial to acceptance--is that [5] is characterized by a modal aspect, such as a *necessary link* (logical or causal) between antecedent and consequent, which seems to be missing in [4]. So, the distinction between indicative and counterfactual conditionals is unquestionably pointed out by this example, at the expense of those aspiring to a unified theory simply denying this difference.[2]

[1] Ernest W. Adams, "Subjunctive and Indicative Conditionals", *Foundations of Language* 6, no. 1 (1970): 89–94.

[2] "Therefore there really are two different sorts of conditional; not a single conditional that can appear as indicative or as counterfactual depending on the speaker's opinion about the truth of the antecedent." David Lewis, *Counterfactuals* (Oxford: Blackwell, 1973), 3.

A clarification must be done in order to explain why counterfactual conditionals are identified with subjunctives and non-counterfactuals with indicatives, although there is not a complete coincidence of these concepts.

First of all, a statement is told "counterfactual" when its antecedent evinces an opposite hypothesis to reality and "subjunctive" when there is, according to English grammar, just a "would" in the main clause and a past tense in the if-clause. It can happen that sometimes these different properties--interpretative and morphological--do not coexist at all, so that some subjunctive conditionals do not exclude the possibility of a true antecedent:

[6] "If Chris went to the party this evening, and she probably will go, Tom would be enthusiastic."

In the same way, it may be possible to use indicative conditionals even if we know the antecedent is false:

[7] "If he is handsome, then I am Naomi Campbell!"

However, many philosophers hold it would be wrong to describe a counterfactual merely as a conditional whose antecedent is false. Rather, it would be better to identify it as a proposition that *invokes* in some way the antecedent's falsity.[3] Indeed when we say:

[8] "If Jones were present at the meeting, he would vote for the motion."

instead of:

[9] "If Jones is present at the meeting, he will vote for the motion."

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[3] "It is not their [the antecedent's and consequent's] falsity in fact that puts a 'counterfactual' conditional into this special class, but the user's expressing in the form of words he uses, his belief that the antecedent is false." John L. Mackie, *Truth, Probability, and Paradox* (Oxford: Clarendon Press, 1973), 71.

"'Counterfactual' may seem to be less open to objection. What lies behind this piece of terminology is not, of course, that the antecedent is in fact false, but that, in some way, the falsehood of the antecedent is implied, whether the conditional is true or false, well supported or not." Michael Woods, David Wiggins, and Dorothy Edgington, *Conditionals* (Oxford: Clarendon Press, 1997), 5.

we are pointing out an information rather than another one: with [8] the speaker wants to focus the attention on what Jones would do if he were present at the meeting--without exclude the fact that he could *not* be present (so *invoking the antecedent's falsity*)--, instead in [9] it is not important that part of the content about Jones' presence (or absence) but, rather, the information concerning the fact he intends to vote for the motion.

Let me present another example:

[10] "If I went to the prom, would you come with me?"

[11] "If I go to the prom, will you come with me?"

In front of these two inferences, the first thought is that who is saying [10] is trying a manner to invite me to the prom--he is saying he would like to go to the prom with me. Instead, about [11] I could simply think that the guy (maybe a neighbor) is offering me just a ride to the party (maybe by car)--I should be totally self-confident to think this inference means a romantic date.

In other words, with [8] and [10] we want to remark just that necessary link between antecedent and consequent characterizing, as formerly said, the counterfactual conditionals rather than the indicative ones. Therefore, if we do not strictly denote counterfactuals with those conditionals whose antecedent is false, [8] and [10] could be easily considered as counterfactuals as the following conditionals:

[12] "If Jones had been present at the meeting, he would have voted for the motion."

[13] "If I had gone to the prom, would you have come with me?"

Furthermore, if we considered only [12] as a counterfactual but not [8] and [10], we should consequently treat the last ones such as contrary-to-facts. In this way we would end to confuse the two well-defined classes. Their differences must be quite established and a type of conditional does not have to work as a supporter for the other one.[4] This idea is entirely shown by examples [4] and [5]: people who accept [4] hardly hold [5]. Instead, a person could easily accept both [8] and [12] recognizing in them the same counterfactual conditional in two different times.[5]

So, in order to facilitate, many philosophers--and I agree--have decided to deal with, in general, subjunctive conditionals as counterfactuals and indicative conditionals as non-counterfactuals.

After this brief introduction I shall define the content of my work. Given that the literature about conditional statements is vast and the topic has been analyzed by different point of view, I prefer not to present a simple list of all theories, but rather to focus the attention on some developments from the late 1960s.[6]

I will dedicate the first chapter to Frank Ramsey and to the different suppositional theories born as interpretations of a piece of his writing--and footnote related--appearing in Ramsey 1929. One consists in a probabilistic reformulation, known as "Equation", that is central in the conditional debate. Many philosophers and logicians advanced several proofs in support or against the Equation. A very important contribute is that of Adams, who had the worth of extending probabilistic logic to conditionals. I will report in which Adams' Hypothesis consists.

The probabilistic thesis of Adams is not the only suppositional theory advanced. Indeed, philosophers like Mackie, Gärdenfors, Harper and Levi (who complements

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[4] "As has been recognized, what would count as strong, or conclusive, support for a non-counterfactual conditional would not support the corresponding counterfactual." Woods et al., *Conditionals*, 7.

[5] Surely, an indicative conditional could become counterfactual with the time, but this is not a proper distinguishing feature, such as examples [4] and [5] shows–neither [4] correspond to [5] or it is [5] in a second moment. At most the indicative "If Oswald didn't kill Kennedy, someone else did" could correspond to some kind of counterfactual like "If Oswald hadn't killed Kennedy, Kennedy would be still alive".

[6] A selective account about the problem of conditionals in the History could be find in David H. Sanford, *If P, then Q: conditionals and the foundations of reasoning* (London: Routledge, 1989).
About the material conditional, I will present Edgington's argument, showing "⊃" such as a no good candidate for *"If A, C"*: 64--68. See also Dorothy Edgington, "On Conditionals", *Mind* 104, no. 414 (1995): 235--329.

Gärdenfors's work) provided an approach to conditionals in terms of non-probabilistic Belief Revision, based on the general idea that an epistemic function *, for a given state of belief *K* and an epistemic input *p*, produces a new suppositional state *K*p*. I will show in which this kind of epistemic logic consists and I will present the famous AGM logic.

The second chapter will present Stalnaker's analysis. It involves the concept of "possible world": the ontological analogue of a stock of hypothetical beliefs allowing the transition from belief conditions to truth conditions. Stalnaker's analysis is known as "semantics of possible worlds" and it was developed independently of Lewis. I will show such a semantic and the fundamental differences between this one and Lewis' thesis.

Semantics of possible worlds, even working in accordance with Adams' hypothesis about simple conditionals, yields some problems in presence of compound conditionals. Additionally, a result of Lewis showed that, if the probability of conditionals is the conditional probability $P(q\,|\,p)$ (as Adams guesses) and if the probability of a proposition is always the probability it is true (as Stalnaker supports), then any proposition *"If p, q"* whose probability of truth coincides with $P(q\,|\,p)$ does not exist unless trivializing the Equation. This is the famous Triviality Result to which I prefer dedicate a separate chapter.

So, the third chapter will show the development of the Triviality Result and its implications, like the incompatibility between Adams and Stalnaker's accounts. So, if Stalnaker firstly agreed with the idea the probability of a conditional equals conditional probability, in front of Lewis' Result he seems to give up the Equation and, in general, a suppositional view. Alternatively, Adams kept on holding his thesis inviting to consider conditionals, not as standard propositions, but as particular linguistic constructions lacking of truth conditions. For this reason Adams treated conditionals only in terms of assertability. His thesis met a lot of supporter especially thanks to Edgington, who presented strong arguments making Adams' logic one of the best candidate for indicative conditional's treatment.

Though not without difficulty, Adams' approach represents one of the most successful theories of conditionals. But the question I raise is this:

Does the Triviality Result lead necessarily to Adams' conclusion to deny any kind of truth conditions and values for indicative conditionals?

In other words:

Might it exist an alternative way to avoid Lewis' Result in accordance with the Equation?

In order to answer to such a question, the fourth chapter will introduce the logic of de Finetti, a kind of three-valued logic called "Logic of Trievents". This seems to avoid the Triviality Result, but it is definitely no free from trouble. However, some limit seems to be overcome by a modified trievents approach, developed by Alberto Mura.

I will dedicate the last chapter to Mura's propose, presented firstly as "Semantics of Hypervaluations" and then improved as "Theory of Hypervaluated Trievents". Mura gave a modified account of de Finetti's trievents--escaping different arguments against the original trievents--with the intent of providing a new semantic for Adams' conditional logic. I will claim that Mura's account can be a good candidate for a semantic of indicative conditionals, in perfect harmony with Adams analysis. Indeed, the Theory of Hypervaluated Trievents incorporates Adams' p-entailment, allowing an extension of it for all trievents--including compound conditionals. In this way we are no more obligated to reject any truth conditions for conditionals.

Moreover, Mura proposed a generalization of the Theory of Hypervaluated Trievents able to catch counterfactual conditionals by introducing a new variable K representing the corpus of total beliefs.

In conclusion, this work wants to put in evidence that conditional issue is not closed and that different additional ways can be investigated. For example, the Theory of Hypervaluations could be a good solution that deserves to be inquired.

Indeed, figuring out the limit of hypervaluations about counterfactuals we can also aspire to a unified theory for conditional sentences.

I

INTERPRETATIONS OF RAMSEY'S TEST

1. Via Bayesian conditionalization: the Equation

Let us consider the famous remark in Ramsey 1929:

"Now suppose a man is in such a situation. For instance, suppose that he has a cake and decides not to eat it, because he thinks it will upset him, and suppose that we consider his conduct and decide that he is mistaken. Now the belief on which the man acts is that if he eats the cake he will be ill, taken according to our above account as a material implication. We cannot contradict this proposition either before or after the event, for it is true provided the man doesn't eat the cake, and before the event we have no reason to think he will eat it, and after the event we know he hasn't. Since he thinks nothing false, why do we dispute with him or condemn him?

Before the event we do differ from him in a quite clear way: it is not that he believes *p*, we $\bar{p}$; but he has a different degree of belief in *q* given *p* from ours; and we can obviously try to convert him to our view.[1] But after the event we both know that he did not eat the cake and that he was not ill; the difference between us is that he thinks that if he had eaten it he would have been ill, whereas we think he would not. But this is *prima facie* not a difference of degrees of belief in any proposition, for we both agree as to all the facts.

(Footnote)

[1] If two people are arguing ' If *p* will *q*? ' and are both in doubt as to *p*, they are adding *p* hypothetically to their stock of knowledge and arguing on that basis about *q*; so that in a sense ' If *p*, *q* ' and ' If *p*, $\bar{q}$ ' are contradictories. We can say they are fixing their degrees of belief in *q* given *p*. If *p* turns out false, these degrees of belief are rendered *void*. If either party believes $\bar{p}$ for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses."[7]

The procedure for evaluating conditional sentences described in this text is called "Ramsey's Test". It inspired a suppositional analysis for conditionals, where "If *p*, *q*" is interpreted as a hypothetically supposition that the antecedent *p* holds the believability of the consequent *q* under that supposition.

Ramsey's Test has been interpreted in different ways. Undoubtedly, the most famous and argued interpretation is that advanced by some philosophers--like

[7] Frank P. Ramsey (1929), "General Propositions and Causality", in *Foundations of mathematics and other logical essays*, ed. R. B. Braithwaite (London: Routledge & Kegan, 1931), 246--247.

Adams and followers--who focused their attention on the concept of "degree of belief"[8], analyzing the remark above in accordance with Ramsey's probability theory--called "logic of partial belief" by Ramsey himself.[9]

Ramsey held that human beliefs cannot be based on an objective theory because they are connected to a whole set of epistemic attitudes--through which people evaluate, choose and act.[10] In this way Ramsey did not mean to deny the existence of objective beliefs, but just to suggest to interpret human knowledge in terms of partial beliefs able to change in front of new evidences. The logic of partial belief

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[8] In Ramsey 1926 there is a full paragraph in which the degree of partial belief is defined:

"The old-established way of measuring a person's belief is to propose a bet, and see what are the lowest odds which he will accept. This method I regard as fundamentally sound; but it suffers from being insufficiently general, and from being necessarily inexact. It is inexact partly because of the diminishing marginal utility of money, partly because the person may have a special eagerness or reluctance to bet, because he either enjoys or dislikes excitement or for any other reason, e.g. to make a book. […]

In order therefore to construct a theory of quantities of belief which shall be both general and more exact, I propose to take as a basis a general psychological theory, […]. I mean the theory that we act in the way we think most likely to realize the objects of our desires, so that a person's actions are completely determined by his desires and opinions. […]

Let us call the things a person ultimately desires ' goods ', and let us at first assume that they are numerically measurable and additive. […]

It should be emphasized that in this essay good and bad are never to be understood in any ethical sense but simply as denoting that to which a given person feels desire and aversion.

The question then arises how we are to modify this simple system to take account of varying degrees of certainty in his beliefs. I suggest that we introduce as a law of psychology that his behavior is governed by what is called the mathematical expectation ; that is to say that, if p is a proposition about which he is doubtful, any goods or bads for whose realization p is in his view a necessary and sufficient condition enter into his calculations multiplied by the same fraction, which is called the ' degree of his belief in p '. We thus define degree of belief in a way which presupposes the use of the mathematical expectation." Frank P. Ramsey (1926), "Truth and Probability", in *Foundations of mathematics and other logical essays*, 172--174.

[9] I personally agree with the suggestion to analyze this passage taking into account Ramsey's philosophy.

[10] An analogous idea was developed by de Finetti. He presented contemporaneously but independently the same philosophy of probability advanced by Ramsey, and published in the same year of Ramsey's posthumous considerations in: Bruno de Finetti (1931), "Sul significato soggettivo della probabilità", in *Fundamenta Mathematicae*, vol. 17, 298--329.

wants to be just a way to calculate our beliefs such as subjective probabilities, establishing Bayes' theorem as the general rule to determine the probability update.

Ramsey's approach for defining probability laws in terms of degrees of belief made some philosophers consider Ramsey 1929 as the application of probability logic--that according to Ramsey is a logic of partial belief--to conditional sentences. So, they interpreted Ramsey's Test via classical Bayesian conditionalization, inviting to measure the probability of "If *p*, *q*" by *conditioning on p*, that is identifying the probability of a conditional with the *conditional probability* on *q* given *p*. This represents the reformulation of Ramsey's Test in a probabilistic thesis known as "Equation":

$P(p \longrightarrow q) = P(q \mid p)$ [where $P(p) > 0$][11]

To properly understand what conditional probability and conditioning are it would be appropriate to make some references to Thomas Bayes, who was able to found an updating rule establishing how to adjust our degree of belief when we acquire new information. This law says the probability of any event *b*, after learning that *a* is true (and nothing else), may be changed. How? The rule prescribed in Bayesian literature is to match the *posterior* probability of *b* ($P_{t1}(b)$) with the *prior* probability of *b* given *a* ($P_{t0}(b \mid a)$). This is just the *Bayesian conditionalisation*--where $P_{t0}(b \mid a)$ is told *conditional probability*--and it can be formulated in this way:

If $P_{t0}(a) > 0$, then $P_{t1}(b) = P_{t0}(b \mid a)$[12]

A simple definition of conditional probability is given in a Bayes' posthumously published work:

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[11] P(p)>0 because of *zero-intolerance* property of conditionals, according to whom if *p* has no chance of being true, there is not any conditional probability. In other words, nobody use a conditional sentence when know that the antecedent's probability is 0. Jonathan Bennett, *A Philosophical Guide to Conditionals*, (Oxford: Clarendon Press, 2003), 53--57.

[12] Richard Jeffrey offered a generalization of this rule that works when we are not totally sure about *a*: $P_{t1}(b) = P_{t0}(b|a)\ P_{t1}(a) + P_{t0}(b|\sim a)\ P_{t1}(\sim a)$. Richard C. Jeffrey, *The Logic of Decision*, 2nd ed. (Chicago: The University of Chicago Press, 1983), 169.

"If there be two subsequent events, the probability of the second b/N and the probability of both together P/N, and it being first discovered that the second event has also happened, from hence I guess that the first event has also happened, the probability I am right is P/b."[13]

Therefore, the probability that both an event e and an hypothesis h will happen, is given by multiplication of the probability of e ($P(e)$) by that of h under the supposition that e occurs ($P(h \mid e)$):

$$P(h \wedge e) = P(h \mid e) \cdot P(e)$$

So the conditional probability is expressed in this ratio:

$$P(h|e) = \frac{P(h \wedge e)}{P(e)}$$ [where both terms exist and $P(e) > 0$]

From this definition it derives the well-known Bayes' Theorem, which has the merit to have inversely related the probability of a conditional hypothesis given some evidence ($P(h \mid e)$) and the probability of the conditional evidence on that hypothesis ($P(e \mid h)$):

$$P(h|e) = \frac{P(e|h) \cdot P(h)}{P(e)}$$ [where $P(h) > 0$ and $P(e) > 0$][14]

Who accepts the Equation identifies it as the rule for calculating the probability of a conditional sentence. So, in whatever way we interpret the connective "⟶" its probability must correspond to $\frac{P(e|h) \cdot P(h)}{P(e)}$, otherwise the Equation is rejected. For instance, because the probability of the material conditional "⊃" is usually different

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[13] Thomas Bayes, "An Essay Toward Solving a Problem in the Doctrine of Chances", *Philosophical Transactions of the Royal Society of London*, vol. 53 (1763): 381.

[14] $P(h|e)$ is the conditional probability of h given e--also known as posterior probability of h in front of a new evidence e. $P(h)$ is the prior probability of h, that is the degree of belief in an hypothesis h--before knowing any information about e. $P(e)$ is the prior probability of e and--unlike $P(e|h)$--it does not take into account any information about h. $P(e|h)$--posterior probability of e given h--is told likelihood of e given fixed h.

Anyway, this is the simplest form of the theorem, derivable by conjunction rule. However, other versions of it exist. See Tim McGrew, "Eight versions of Bayes's theorem" (2005): http://homepages.wmich.edu/~mcgrew/Bayes8.pdf from the conditional probability, the supporters of the Equation exclude that "⟶" can be "⊃".

Many arguments have been advanced pro and against the Equation, occupying a central role into the debate about conditionals that characterized the whole second part of 20th century.

2. Adams' thesis

Ernest Adams is one of the first supporters of the Equation. Indeed Bennett wrote:

"This powerful, simple, and attractive thesis was first made widely known by Stalnaker (1970), and it has been called 'Stalnaker's Hypothesis'. But he tells me that Ernest Adams and Richard Jeffrey propounded it before he did; and Adams says that Brian Ellis deserves equal credit; so I shall leave personal names out of it, and follow Edgington in calling it the Equation."[15]

Since mid-90s, Adams showed powerful arguments defending the probabilistic interpretation of Ramsey's Test, so that some philosophers started to talk about Ramsey-Adams Thesis.

Adams' analysis is restricted to *indicative* conditionals and started observing that the standard use of propositional calculus leads to fallacies when its application involves conditional sentences. So, the problem Adams raised concerns how we have to use formal logic in conditional treatment. Indeed, he showed that a lot of cases[16] which are classically valid--in the sense that it is impossible for the premises to be true while its conclusion is false--are rejected (or at least doubtful) by the common sense, leading to different kinds of fallacies.

Adams identified the trouble with the fact that when we deal with conditional statements, the term "true" has a no so clear application. For this reason, he proposed to find a kind of validity that does not involve the notion of truth, with the

[15] Jonathan Bennett, *A Philosophical Guide to Conditionals*, 57.

[16] In Adams 1965 we can found 9 fallacies about the application of propositional calculus. See Ernest W. Adams, "The Logic of Conditionals", *Inquiry: An Interdisciplinary Journal of Philosophy*, vol. 8 (1965): 166--197.

intent to analyze conditional sentences from a point of view and not in terms of their truth conditions. So, he substituted the concept of classical validity with that of "reasonableness", whose condition is:

"If an inference is reasonable, it should not be the case that on some occasion the assertion of its premises would be justified, but denial of its conclusion also justified"[17].

So, while classical validity involves the notion of true, the reasonableness concerns that one of *justified assertability*, which is not a construct typically mathematical or scientific but rather a concept whose content is dictated by the context of the assertion. Indeed, an assertion of a statement is justified if and only if *what is known* on that occasion gives us either the certainty or the probability that the same statement will be true and a *bet* on it will be won. In the same way, the denial of that assertion is justified if and only if we have either the certainty or the probability that the statement will be false and the bet will be lost. Adams called the assertion *strictly justified* in case of certainty and *probabilistically justified* when the statement is just probable.

What about the assertion of "If *p*, *q*"? Adams converted the above notions in terms of conditional bets[18]--any bets on conditional statements--giving such a "betting" criterion of justification:

"a. The assertion of a bettable conditional 'if *p* then *q*' is strictly justified on an occasion if what is known on that occasion makes it certain that either *p* is false or *q* is true; its denial either *p* is false or *q* is false.

b. The assertion of a bettable conditional 'if *p* then *q*' is probabilistically justified on that occasion if what is known on that occasion makes it much more likely that *p* and *q* are both true than that *p* is true and *q* is false; its denial is probabilistically

[17] Ernest W. Adams, "The Logic of Conditionals", 171.

[18] The notion of conditional bet was introduced first by de Finetti 1931 in: Bruno de Finetti, "Sul significato soggettivo della probabilità", Fundamenta Mathematicae (1931): 298--329. See also Bruno de Finetti, "La Prévision: Ses Lois Logiques, Ses Sources Subjectives", *Annales de l'Institut Henri Poincaré* 7 (1937), 1--68. Translated as "Foresight: Its Logical Laws, Its Subjective Sources", in *Studies in Subjective Probability,* eds. H. E. Kyburg Jr. and H. E. Smokler (New York: Robert E. Krieger, 1980).

justified if it is much more likely that *p* is true and *q* is false than that *p* and *q* are both true.

c. (Definition) The assertion and denial of a bettable conditional 'if *p* then *q*' are both vacuously probabilistically and strictly justified on an occasion if what is known on that occasion makes it certain that *p* is false."[19]

Conditions for reasonableness of conditionals	$p \rightarrow q$	$\sim(p \rightarrow q) \equiv p \rightarrow \sim q$
Strictly justified	$\sim p \lor q \equiv P(\sim p \lor q)=1$	$\sim p \lor \sim q \equiv P(\sim p \lor \sim q)=1$
Probabilistically justified	$P(p \land q) > P(p \land \sim q)$	$P(p \land \sim q) > P(p \land q)$
Vacuously strictly and probabilistically justified	$\sim p \equiv P(p)=0$	$\sim p \equiv P(p)=0$

In case of vacuously justification, the inference and its denial may be asserted as well because the bet is not lost but just *called off*--according to the betting criterion. However, Adams pointed out that when we are sure the bet will be called off we will not stake at all and no indicative conditional is actually asserted. Indeed, in those cases in which we are sure about antecedent's falsity we will use a subjunctive conditional--no object of Adams' analysis.

Taking into account the notion of vacuous conditional, Adams reformulated the general condition for reasonableness of an inference saying that *it cannot be the case the assertion of its premises and the non-vacuous denial of its conclusion are both justified on the same occasion.*

In view of this notion of reasonableness, Adams showed that absurd cases classically valid--for example, the material conditional's fallacies--are not as much

[19] Ernest W. Adams, "The Logic of Conditionals", 176--177.

valid for the betting criterion of justification. Indeed, inferences like "If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life"[20] are classically valid but not *reasonable*, because both the assertion of the premises and the negation of the conclusion are justified.

In Adams 1965, we can find an informally presentation of a reasonableness' criterion using the standard probability calculus. Adams concluded that this first analysis, though solving several problems in conditional treatment, shows some critical limitation. For example, its application lacks with conditionals derived from suppositions and with compounds involving conditionals. So, he advanced the hypothesis that, maybe, assertable conditionals observe different logical laws, other from those one of the standard propositional calculus.

Trying to overcome these limitations, in Adams 1966[21] the original idea is formalized with some adjustment. First of all, the notion of "justified assertability" is now replaced by that of "high probability", and the criterion of reasonableness is consequently given simply substituting "true" with "high probability" in the definition of classical validity:

"an inference is *reasonable* just in case it is impossible for its premises to have high probability while the conclusion has low probability"[22].

Then, in Adams 1975, a consequence of this assumption is made explicit, introducing a technical term called "uncertainty" (u = 1– probability). So, in case of reasonable inference:

"[…] *the uncertainty of its conclusion cannot exceed the uncertainty of its premises* (where uncertainty is here defined as probability of falsity […])".[23]

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[20] Ernest W. Adams, "The Logic of Conditionals", 166.

[21] Ernest W. Adams, "Probability and the Logic of Conditionals", *in Aspects of Inductive Logic*, eds. J. Hintikka and P. Suppes (Amsterdam: North Holland Publishing Co., 1966), 265--316.

[22] Ernest W. Adams, "Probability and the Logic of Conditionals", 266.

[23] Ernest W. Adams, *The Logic of Conditionals. An Application of Probability to Deductive Logic* (Dordrecht: Synthese Library, D. Reidel Publishing Co., 1975), 2.

The concept of uncertainty is fundamental in Adams because through it he could totally avoid any concept of falsity in the definition of validity.

Therefore, in Adams's hypothesis the strict connection between high probability and truth, characterizing unconditional statements, instead lacks for conditionals[24]. Indeed, Adams advanced the idea according to which the probability of a conditional statement should not be interpreted as *probability of being true* but rather as *conditional probability*. Thus, Adams identified the Equation as a fundamental assumption of his analysis, making his thesis one of the most important arguments in defense of the Equation itself.

A complete formal presentation of his theory can be found in Adams 1975. Let me give a summary:

- Syntactical concepts and terminology:
  - *Factual language* $\mathscr{L}$: language generated by any set of sentential variables (capital letter like *"A"*, *"B"*, etc.) together with the two sentential constant *"T"* and *"F"* (for logical truth and falsehood respectively). It is a *sublanguage* of another if all of its formulas are also formulas of the other one. It is a *finite* language if it contains a finite number of atomic formulas.
  - *Factual formulas* (lowercase Greek letters like *"ϕ"*, *"ψ"*, *"η"*, etc.): formulas of a factual language.
  - *Conditional formulas* of $\mathscr{L}$: every expression of the form "$\phi \Rightarrow \psi$", where the antecedent ϕ and the consequent ψ are formulas of $\mathscr{L}$ and ϕ is not false. The connective "⇒" occurs just as a main connective in a formula (the antecedent and the consequent are not conditional formulas themselves).

[24] "The probability of a proposition is the same as the probability that it is true. [...] What we want to argue next is that there is a much more radical divergences between the two soundness criteria in application to inferences involving conditional propositions, which is ultimately traceable to the failure of the probability equals probability of truth assumption in application to conditionals." See Ernest W. Adams, *The Logic of Conditionals. An Application of Probability to Deductive Logic*, 2.

- *Conditional extension:* the set of all formulas of $\mathscr{L}$ together with all conditional formulas. The conditional extension is called *conditional language* and $\mathscr{L}$ is its *factual basis*. Both factual and conditional formulas of a conditional language are represented by script capitals variables like "$\mathscr{A}$", "$\mathscr{B}$", etc., while capital letters like *"X", "Y"*, etc., stand to the sets of these formulas.
- *Conditionalization* of a factual formula ϕ: $T \Rightarrow \phi$. A factual formula and its conditionalization are *probabilistically equivalent*.
- *Material counterpart* of a conditional formula $\phi \Rightarrow \psi$: the factual formula $\phi \supset \psi$. If the antecedent ϕ is non-logically true, then the material counterpart $\phi \supset \psi$ is never probabilistically equivalent to $\phi \Rightarrow \psi$.
- *Contrary* of a conditional: $\sim(\phi \Rightarrow \psi) = \phi \Rightarrow \sim\psi$.
- *Quasi-conjunction* of a finite non-empty set of conditional formulas $X = \{\phi_1 \Rightarrow \psi_1, \dots, \phi_n \Rightarrow \psi_n\}$: $C(X) = (\phi_1 \vee \dots \vee \phi_n) \Rightarrow [(\phi_1 \supset \psi_1)\ \&\ \dots$ $\&\ (\phi_n \supset \psi_n)]$.
- *Quasi-disjunction* defined for finite sets X of conditional formulas: $D(X) = (\phi_1 \vee \dots \vee \phi_n) \Rightarrow [(\phi_1\ \&\ \psi_1) \vee \dots \vee (\phi_n\ \&\ \psi_n)]$.

* Truth-conditional semantics:
  - *Truth-assignment* for a factual language $\mathscr{L}$: a function t which fixes a value of truth or falsity for the formulas of $\mathscr{L}$, so that $t(T)$=1 and $t(F)$=0. Regarding formulas like $\phi \supset \psi$, $t(\phi \supset \psi)$ is 1 if and only if either $t(\phi)$=0 or $t(\psi)$=1, so that "⊃" is the material conditional. A finite language has a finite number of different t.
  - *State-description:* a formula ϕ_t corresponding to every t of a finite language such that, for any factual formula ϕ of $\mathscr{L}$, $E(\phi)$=1 if and only if ϕ is logically consistent with ϕ_1.
  - *Verification* of a conditional formula $\phi \Rightarrow \psi$ under t: if $t(\phi)=t(\psi)$=1. Its material counterpart is also verified under t if and only if the conditional is not falsified (although $\phi \Rightarrow \psi$ could be not verified either) under t. If $\phi \Rightarrow \psi$ is verified, its contrary $\phi \Rightarrow \sim\psi$ is falsified.

- *Falsification* of a conditional formula $\phi \Rightarrow \psi$ under *t:* if $t(\phi)=1$ and $t(\psi)=0$. Its material counterpart is also falsified under *t* if and only if the conditional is falsified under *t*. If $\phi \Rightarrow \psi$ is falsified, its contrary $\phi \Rightarrow \sim\psi$ is verified.
- *Neither verification nor falsification* of a conditional formula $\phi \Rightarrow \psi$ under *t:* if $t(\phi)=0$.
- *Verification and falsification* of a factual formula ϕ under *t:* by identifying it with its conditionalization $T \Rightarrow \phi$, so that ϕ is verified or falsified when $t(\phi)$ is 1 or 0. A factual formula cannot be neither verified nor falsified.
- *Verification* of a quasi-conjunction $C(X)$ under *t:* if *no* member of X is falsified and *at least one* is verified under *t*. In this case X is *confirmed* by *t*. Generally, X is *confirmable* if there exists a truth-assignment confirming it.
- *Falsification* of a quasi-conjunction $C(X)$ under *t:* if *at least one* member of X is falsified under *t*.
- *Neither verification nor falsification* of a quasi-conjunction $C(X)$ under *t:* if *no* member of X is neither verified nor falsified under *t*.
- *Verification* of a quasi-disjunction $D(X)$ under *t:* if *at least one* member of X is verified under *t*.
- *Falsification* of a quasi-disjunction $D(X)$ under *t:* if *no* member of X is verified and *at least one* is falsified under *t*. In this case X itself is *disconfirmed* by *t*. Generally, X is *disconfirmable* if there exists a truth-assignment confirming it.
- Some *verification equivalency:* $\sim(\sim\mathscr{A}) \equiv \sim\mathscr{A}$; $C(\sim X) \equiv \sim D(X)$; $D(\sim X) \equiv \sim C(X)$. *t* confirms X if and only if it disconfirms $\sim X$ and *t* disconfirms X if and only if it confirms $\sim X$.

- Probability definitions:
  - *Probability-assignment* of a factual language $\mathscr{L}$*:* a function P assigning, for every formulas of $\mathscr{L}$, a real number between 0 and 1 so satisfying the Kolmogorov Axioms according to which if ϕ logically entails ψ then

$P(\phi) \leq P(\psi)$ and $P(T)=1$, and if ϕ and ψ are logically inconsistent then $P(\phi \vee \psi) = P(\phi) + P(\psi)$. A probability-assignment is a truth-assignment only in case it assigns one of the two extreme values (1 and 0). If the language is finite, the probability of any proposition ϕ equals the sum of the probabilities of the truth where it would be true: $P(\phi) = P(t_1)t_1(\phi) + \ldots + P(t_n)t_n(\phi)$. If P is a probability function for $\mathscr{L}$, and $\mathscr{L}$ is a sublanguage of $\mathscr{L}'$, then there is a probability function P' for $\mathscr{L}'$ such that: $P'(\phi) = P(\phi)$ for all ϕ in $\mathscr{L}$.

- *Uncertainty* of a factual formula ϕ of $\mathscr{L}$ relative to P: $u_p(\phi) = P(\sim\phi) = 1 - P(\phi)$, that is the number measuring the degree to which ϕ is considered unlikely. If ϕ entails ψ, then $u_p(\psi) \leq u_p(\phi)$ and $u_p(\phi_1 \& \ldots \& \phi_n) \leq u_p(\phi_1) + \ldots + u_p(\phi_n)$.
- A probability-assignment is *proper* for a conditional formula $\phi \Rightarrow \psi$ if $P(\phi) \neq 0$, and it is proper for X if it is proper for all conditional formulas of X.
- *Conditional probability* of a conditional formula $\phi \Rightarrow \psi$ relative to a probability-assignment P for $\mathscr{L}$: $\frac{P(\phi \,\&\, \psi)}{P(\phi)}$.
- *Uncertainty* of $\phi \Rightarrow \psi$ relative to P: $1 - P(\phi \Rightarrow \psi)$.
- Some properties of conditional probability and uncertainty: $P(\phi) = P(T \Rightarrow \phi)$; if ϕ is the material counterpart of $\mathscr{A}$, $P(\mathscr{A}) \leq P(\phi)$; $P(\sim\mathscr{A}) = 1 - P(\mathscr{A})$; $P(\sim D(X)) = P(C(\sim X))$; $P(\sim C(X)) = P(D(\sim X))$; $P(D(X)) \leq$ the sum of the probabilities $P(\mathscr{B})$ for $\mathscr{B}$ in X; $u_p(C(X)) \leq$ the sum of the uncertainties $u_p(\mathscr{B})$ for $\mathscr{B}$ in X. Properties $P(\phi) = P(T \Rightarrow \phi)$ and $P(\sim D(X)) = P(C(\sim X))$ entail that $P(\phi)$ and $P(T \Rightarrow \phi)$, $\sim D(X)$ and $C(\sim X)$, $\sim C(X)$ and $D(\sim X)$ are *probabilistically-equivalent*. If two formulas are verification-equivalent, they are also probabilistically-equivalent.
- General rule which follows from the generalization of $P(\phi) = P(t_1)t_1(\phi) + \ldots + P(t_n)t_n(\phi)$:

$$P(\phi \Rightarrow \psi) = \frac{P(t_1)t_1(\phi\&\psi) + \cdots + P(t_n)t_n(\phi\&\psi)}{P(t_1)t_1(\phi) + \cdots + P(t_n)t_n(\phi)}$$

where t_i are the truth-assignment for $\mathscr{L}$.

This rule says that the probability of a conditional formula equals the ratio between the probability of its being verified and the probability of its being either verified or falsified.

- Probability consistency:
  - [DEFINITION 1]. Let $\mathscr{L}$ be a factual language and let X be a set of formulas of its conditional extension. X is *probabilistically-consistent* (p-consistent) if and only if for every real number $\varepsilon > 0$, there exists a probability-assignment p for $\mathscr{L}$ which are proper for X such that $P(\mathscr{A}) \geq 1 - \varepsilon$ for all $\mathscr{A}$ in X.
  - [THEOREM 1]. Let $\mathscr{L}$ be a factual language, let p be a probabilistically assignment for $\mathscr{L}$, and let X be a finite set of formulas of the conditional extension of $\mathscr{L}$ such that p is proper for X.
    1.1. If there exists a non-empty subset of X which is not confirmable, then the sum of the uncertainties $u_p(\mathscr{A})$ for $\mathscr{A}$ in X is at least 1, and hence X is not p-consistent.
    1.2. If every non-empty subset of X is confirmable then X is p-consistent.
  - [THEOREM 2]. Let $\mathscr{A}$ be a factual or conditional formula, let X be a finite set of such formulas, and let X' be the set of material counterparts of formulas in X.
    2.1. If X' is logically inconsistent than X is probabilistically inconsistent (p-inconsistent).
    2.2. If X is p-inconsistent and contains at least one factual formula and no proper p-inconsistent subsets of X, then X' is logically inconsistent.
    2.3. If X is p-consistent than either $X \cup \{\mathscr{A}\}$ or $X \cup \{\sim\mathscr{A}\}$ is p-consistent.

- [THEOREM 3]. Let $\mathscr{L}$ be a factual language, let $\mathscr{A}$ and $\mathscr{B}$ be formulas of its conditional extension, and let X be a finite set of such formulas. Let p be a probability-assignment for $\mathscr{L}$ which is proper for $\mathscr{A}$, $\mathscr{B}$ and X. Let $\mathscr{A}'$ be the material counterpart of $\mathscr{A}$ and let X' be the set of all material counterparts of formulas in X.
  - 3.1. If X p-entails $\mathscr{A}$ then $u_p(\mathscr{A})$ is no greater than the sum of the uncertainties $u_p(\mathscr{B})$ for $\mathscr{B}$ in X.
  - 3.2. If X does not p-entail $\mathscr{A}$ then for all $\varepsilon > 0$ there exists a probability-assignment q for $\mathscr{L}$ which is proper for $\mathscr{A}$ and X such that $q(\mathscr{B}) \geq 1-\varepsilon$ for all $\mathscr{B}$ in X, but $q(\mathscr{A}) \leq \varepsilon$.
  - 3.3. If X is p-consistent and p-entails $\mathscr{A}$ then X' logically entails $\mathscr{A}'$.
  - 3.4. If X' logically entails $\mathscr{A}'$ and $\mathscr{A}$ is factual, then X p-entails $\mathscr{A}$.
  - 3.5. X p-entails $\mathscr{A}$ if and only if $X \cup \{\sim\mathscr{A}\}$ is p-inconsistent; X p-entails all formulas if and only if it is p-inconsistent .
  - 3.6. If both $X \cup \{\mathscr{B}\}$ and $X \cup \{\sim\mathscr{B}\}$ p-entail $\mathscr{A}$ then X p-entails $\mathscr{A}$.
  - 3.7. If $\mathscr{A}$ is conditional, X contains at least one factual formula, and X p-entails $\mathscr{A}$ but no proper subset of X p-entails $\mathscr{A}$, then X p-entails both the antecedent and consequent of $\mathscr{A}$.
  - 3.8. If *"A"* is a sentential variable not occurring in X and ϕ is a factual formula, then X p-entails $A \Rightarrow \phi$ if and only if either X is p-inconsistent or $A \supset \phi$ is logically true.

- Probabilistic entailment:
  - [THEOREM 4]. Let $\mathscr{L}$ be a factual language, let ϕ, ψ and η be formulas of $\mathscr{L}$, let $\mathscr{A}$ be a formula of the conditional extension of $\mathscr{L}$, and let X and Y be finite sets of such formulas.
    - 4.1. X p-entails all of its members, and if X p-entails all of members of Y and Y p-entails $\mathscr{A}$, then X p-entails $\mathscr{A}$.
    - 4.2. X p-entails $\mathscr{A}$ if and only if $\mathscr{A}$ is derivable from X using the following seven rules of inference:

[R1]. $T \Rightarrow \phi$ and ϕ are interderivable.

[R2]. If ϕ is logically consistent and ϕ and ψ are logically equivalent then $\phi \Rightarrow \eta$ can be derived from $\psi \Rightarrow \eta$.

[R3]. If ϕ is logically consistent and logically entails ψ, then $\phi \Rightarrow \psi$ can be derived from the empty set.

[R4]. $(\phi \vee \psi) \Rightarrow \eta$ can be derived from $\phi \Rightarrow \eta$ and $\psi \Rightarrow \eta$.

[R5]. If ϕ & ψ is logically consistent then $(\phi \,\&\, \psi) \Rightarrow \eta$ can be derived from $\phi \Rightarrow \eta$ and $\phi \Rightarrow \psi$.

[R6]. $\phi \Rightarrow \eta$ can be derived from $\phi \Rightarrow \psi$ and $(\phi \,\&\, \psi) \Rightarrow \eta$.

[R7]. If ϕ is logically consistent but ϕ & ψ are logically inconsistent, then anything can be derived from $\phi \Rightarrow \psi$.

4.3. Assume that $A_1,\ldots, A_n$ and B are distinct sentential variables of $\mathscr{L}$. There is no set X of formulas of the conditional extension of $\mathscr{L}$ with less than n members which is p-equivalent to the set $\{A_1 \Rightarrow B,\ldots, A_n \Rightarrow B\}$ in the sense that all members of X are p-entailed by this set and X p-entails all members of this set.

- Derived rules:

[R8]. If ϕ logically implies ψ then $\eta \Rightarrow \psi$ can be derived from $\eta \Rightarrow \phi$.

[R9]. $\phi \Rightarrow (\psi \,\&\, \eta)$ can be derived from $\phi \Rightarrow \psi$ and $\phi \Rightarrow \eta$.

[R10]. $\phi \Rightarrow \psi$ can be derived from $\phi \Rightarrow (\psi \,\&\, \eta)$.

[R11]. $(\phi \vee \eta) \Rightarrow (\phi \supset \psi)$ can be derived from $\phi \Rightarrow \psi$.

[R12]. $(\phi_1 \vee \phi_2) \Rightarrow ((\phi_1 \supset \psi_1) \,\&\, (\phi_2 \supset \psi_2))$ is derivable from $\phi_1 \Rightarrow \psi_1$ and $\phi_2 \Rightarrow \psi_2$.

[R13]. C(S) can be derived from S.

[R14]. If ϕ_1 & ψ_1 logically implies ϕ_2 & ψ_2 and $\phi_1 \supset \psi_1$ logically implies $\phi_2 \supset \psi_2$ then $\phi_2 \Rightarrow \psi_2$ can be derived from $\phi_1 \Rightarrow \psi_1$.

[R15]. If $\mathscr{A}$ and $\mathscr{B}$ are factual or conditional and $\mathscr{A}$ p-entails $\mathscr{B}$ then $\mathscr{B}$ can be derived from $\mathscr{A}$.

Let me focus the attention on the reported general rule

$$P(\phi \Rightarrow \psi) = \frac{P(t_1)t_1(\phi\&\psi) + \cdots + P(t_n)t_n(\phi\&\psi)}{P(t_1)t_1(\phi) + \cdots + P(t_n)t_n(\phi)}$$

which represents just a generalization of the idea that probability equals probability of truth. This rule entails that truth-conditional validity ensures reasonableness (*probabilistic-validity*) but, according to Adams, it holds *only* in case of factual propositions. Thus, if Adams' supposition is correct, the probability of a conditional cannot equal *in general* the probability it is true.

The fact that such a link between truth-conditional validity and p-validity holds for factual proposition is easily demonstrable. For example, an inference like "It will either rain or snow tomorrow ($R \vee S$); it will not snow tomorrow ($\sim S$); therefore it will rain tomorrow (R)"[25] is classically valid when $R \vee S$ and $\sim S$ are true, and R is true too. Now, suppose that both $P(R \vee S)$ and $P(\sim S)$ equal 95%, so that both $P(\sim R \& \sim S)$ and $P(S)$ are of 5%. Under these circumstances, the sum of the uncertainties of the premises is of 10%, thus the $u(R) \leq 10\%$. This means that, if the premises have *objective* probabilities of 95%, their conclusion has a probability of at least 90%, and this connection between objective probabilities and correct predictability makes that truth-conditional validity guarantees the probabilistic-validity.

The above-mentioned connection cannot be shown in case of conditional sentences, and the truth-conditional validity lacks to be a proof for reasonableness. For example, considering the conditional inference "If I eat those mushrooms, I will be poisoned"[26]. The simple fact to not eat the mushrooms makes the inference materially true, but it is really difficult to say whether the assertion is right or wrong, so that the decision connected to it would be the best or the worst in terms of practical interest. Indeed, if the mushrooms are not poisoned, but absolutely delicious porcinis, and I decide to not eat them, my choice would not be right. This

[25] Ernest W. Adams, *The Logic of Conditionals. An Application of Probability to Deductive Logic*, 88.

[26] Ernest W. Adams, *The Logic of Conditionals. An Application of Probability to Deductive Logic*, 89.

confirms Adams' intuition, according to which the truth-conditional validity of a conditional inference does not prove its reasonableness--its probabilistic validity cannot be guaranteed by classical validity--so that the rule according to which probability equals probability of true lacks with conditional inferences. Why? The explanation, according to Adams, is that when we assert a conditional we do not express the probability it is true, but rather its conditional probability such as it is presented by Ramsey's Test. This approach should explain a lot of phenomena, like the mushrooms' example, in an easier way than standard probability does. This is the reason for which Adams' Thesis is also known as Probability Conditional Thesis --(PCT): $P(p \rightarrow q) = P(q\,|\,p)$--and its relation with the Material Conditional Thesis--(MCT): $p \rightarrow q = p \supset q = \sim p \vee q$ --is fixed by the Conditional Deficit Formula (CDF)[27]: $P(p \supset q) - P(p \rightarrow q) = [1 - P(p \supset q)][\frac{P(\sim p)}{P(p)}]$.

Even though sometimes--when CDF is low--conditional probability can be inferred by material conditional, such a rule shows why generally they do not coincide at all[28].

The step from the idea that the probability of indicative conditionals is not the probability they are true--and so that $P(p \rightarrow q) \neq P(p \supset q)$--to the conclusion that they have neither truth-values nor, in general, truth conditions seems really obvious. However, I want to point out that Adams definitely denied any truth-values and conditions for indicative conditionals only in 1975, after knowing the problems raised by Lewis' Triviality Result. Of course, Adams proposed to analyze conditional inferences in term of probability since his first approach, but I think this is different from the totally denial of truth conditions. I am not completely sure he would have advanced such a "drastic" solution if any Triviality Result would not have been

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[27] Ernest. W. Adams, "What Is at Stake in the Controversy over Conditional", in Conditionals*, Information, and Inference*, International Workshop, WCII 2002, Hagen, Germany, May 13-15 2002, eds. G. Kern-Isberner, W. Rödder, F. Kulmann (Verlag, Berlin, Heidelberg: Springer, 2005), 1--11.

[28] That these two kinds of probability do not coincide could be shown by a lot of example. One of these could be found in Ernest. W. Adams, "What Is at Stake in the Controversy over Conditional", 1--2.

possible. Also because, even if supporting $P(p \rightarrow q) = P(p \mid q)$ we are refusing the truth conditions of material conditional, it simply means denying the "extreme" opinion according to which conditional inferences *always* have truth-values. Anyway, I will go back to Adams' position--according to me, absolutely pragmatist--after exposing the well-known Triviality Result and its consequences.

Although Adams' logic works pretty well with indicative conditionals, his thesis presents some limits not really holding in natural language. For example, inferences like "If it is sunny, then if it is my day off then I will go to the beach"--$p \rightarrow (q \rightarrow z)$--are excluded by Adams, but they may be asserted ordinarily, equalizing inferences like "If it is sunny and it is my day off, then I will go to the beach"--$(p \wedge q) \rightarrow z$--by the *Law of Importation*[29]. Also inferences joining a standard proposition and a conditional one, like "Either I will stay at home or if Jane calls me then I will go to the cinema"--$p \vee (q \rightarrow z)$--are rejected by Adams, although they are really common in natural language.

However, our language is full of complications to represent a real argument for rejecting a logic that seems to work well under a lot of aspects. Perhaps, also for this reason Adams' hypothesis met several supporters. One of the most important is Dorothy Edgington, whose contribute helped to make Adams' thesis one of the most shared in conditionals' field. Her arguments support either the Equation either Adams' conclusion that accepting $P(p \rightarrow q) = P(q \mid p)$ doubtless means to deny any truth conditions for conditional statements. Her contribution made Adams' hypothesis stronger in front of the Triviality Result, reason for what I prefer presenting Edgington's view after exposing such a result. But now let me keep on showing some other suppositional theories.

[29] Law of Importation: $[p \rightarrow (q \rightarrow z)] \rightarrow [(p \wedge q) \rightarrow z]$. Vann McGee presented an argument supporting the ejection of iterated conditionals, reporting that when we say "If p, then if q then z" we are not accepting an iterated conditional, but rather a conditional with conjunctive antecedent, because what we have in mind is the conditional belief expressed by $(p \wedge q) \rightarrow z$. See Vann McGee, "A Counterexample to Modus Ponens", *The Journal of Philosophy*, Vol. 82, No. 9 (1985): 462--471.

3. Other suppositional theories: Mackie, Harper, Levi, Gärdenfors

Via classical Bayesian conditionalization is not the only possible interpretation of Ramsey's Test. Indeed, some other philosophers read that famous paragraph of Ramsey 1929 in terms of *non-probabilistic belief revision*. In this regard, Gärdenfors developed a semantically theory which found his precursors in Mackie and in those philosophers who elaborated Mackie's writing--like Harper and Levi.

The general analysis offered by Mackie is based on the idea that saying "If *p, q*" we are asserting *q* within the scope of the supposition that *p*. So, given that the primary function of "if" is to introduce a supposition, it could be translated with "suppose that" and it invites us to consider a *possibility*. Clearly, this kind of procedure cannot work with a material conditional, neither with any semantics of possible worlds--because the possibility Mackie has in mind "needs to be explained in terms of concrete human procedures, and not the other way round"[30].

Mackie's suppositional account treats conditional sentences not such as any essentially linguistic reasoning, but in terms of something that goes beyond any language grammatically structured. According to him, in such a way we can explain every standard use of conditionals. Indeed, interpreting a conditional in terms of supposition we can either abandon the idea they are any strict sense statements--simply true or simply false--either understand why sometimes they might assert the corresponding material conditional, sometimes a literally or concrete possible world, and go on.

Now, since a supposition invokes a *possible situation*--while the truth, in a strict sense, belongs to *actual descriptions*--Mackie reject to say conditionals have, in their basic use, a truth-value[31]. Therefore he wrote:

"The general semantic structure of conditionals does not provide them with truth-values in all cases, but in some only; and this would be at least an awkward

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[30] John L. Mackie, *Truth, Probability, and Paradox*, 98.

[31] John L. Mackie, *Truth, Probability, and Paradox*, 105--106.

thing to accommodate in an attempt to display the whole structural semantics of a language in a truth-definition."[32]

A suppositional account should help us to solve some problems with the logic of conditionals, like his application to counterfactuals. Indeed, Mackie said a supposition allows to introduce a different situation from the actual one, so that the conditional could be analyzed in the light of another possibility. This happens because we know the world is governed by causal laws able to produce various effects. Hence, Mackie suggested that:

"Counterfactual conditionals are not to be taken literally as truth about possible worlds, but as a species of human procedure. They are just non-material conditionals plus a hint that their antecedents are unfulfilled, and non-material conditional merely express the asserting of something within the scope of some supposition—which may be done for any one of number of reasons which may themselves be reasonable or unreasonable."[33]

Every argument is built from suppositions to conclusion, suppressing some premises--in quality of modified believes--in order to make the suppositions consistent.

Certainly Mackie's proposal is not free from difficulties--like Edgington pointed out[34]--but, since his approach requires a notion of corrigibility of belief, it caught the attention of those philosophers interested in a suppositional account different from the probabilistic one presented by Adams. Particularly important is Harper and Levi's analysis of Mackie's papers, because they presented an account[35] leading a

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[32] John L. Mackie, *Truth, Probability, and Paradox*, 108.

[33] John L. Mackie, *Truth, Probability, and Paradox*, 114--115.

[34] See a Review by Dorothy Edgington, "Truth, Probability and Paradox: Studies in Philosophical Logic. by J. L. Mackie", *Mind*, New Series, Vol. 85, No. 338 (1976): 303--308.

[35] For information about Harper and Levi's identities, see Zhiqiang Zhuang, *Belief Change under the Horn Fragment of Propositional Logic* (Doctoral dissertation, University of New South Wales, 2013), 16--17. Also see Oliver Schulte, "How do the Harper and Levi Identities Constrain Belief Change?", in *Truth and Probability, Essays in Honour of Hugh LeBlanc* , eds. B. Brown and F. LePage, (London: College Publications at King College London, 2006), 123--137.

correlation between revision and contraction--two fundamental operators in Belief Revision's field[36].

Either Harper either Levi's interpretation of Ramsey's Test is in terms of *minimal revision*, so that to accept a counterfactual conditional means that the minimal revision of knowledge to accept the antecedent requires the acceptance of the consequent too. Thus, they developed, independently, a system for representing rational belief change, allowing any revision of previously accepted evidences. But, while Harper dealt with sentences as proposition, Levi treated them as objects of belief.[37]

The contribution of these philosophers has been fundamental in Belief Revision, particularly in developing one of its most important model, known as "AGM" because of its three inventors, Alchourrón, Gärdenfors and Makinson. They advanced the idea that a belief state is a logically closed (under the rules of deductive logic) set of sentences, called *belief set*. In other word, saying that *p* belong to a belief state *K* means that *p* is a set member of *K*. A belief set designates the agent's view of the world, a *static world* which does not change.[38] It is such a view of the world--the agent's beliefs--that changes, because it is regularly subjected to new information. Hence, the belief set is exposed to an *input*, making that it will be revised and, consequently, creating a new belief set.

AGM is studied in many areas of AI and represents a milestone in Belief Revision. Let me resume its postulates and its approach[39]:

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[36] See Sven Ove Hansson, "Logic of Belief Revision", *The Stanford Encyclopedia of Philosophy* (Fall 2011 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2011/entries/logic-belief-revision/>.

[37] William L. Harper, "Rational Conceptual Change", *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, Volume Two: Symposia and Invited Papers (1976): 462--494.

[38] Zhiqiang Zhuang, *Belief Change under the Horn Fragment of Propositional Logic*, 10.

[39] For a first approach to AGM theory, I suggest Horacio Arló-Costa, Arthur. P. Pedersen, "Belief Revision", in *Continuum Companion to Philosophical Logic*, eds. L. Horsten and R. Pettigrew, (London: Continuum Press, 2011).

- Preliminaries and terminology:
  - *Belief-representing sentences* (lower-case letters like *"p", "q",* etc.): sentences representing beliefs in some propositional language--an AGM language formed by those sentences, elements of it. Actually, sentences do not capture every aspect of belief, but they are considered the best candidate for this purpose. The language contains usual truth-functional connectives and symbols like $\perp$ and $\top$ denoting, respectively, an arbitrary contradiction and an arbitrary tautology.
  - *Set of belief-representing sentences* (capital letters like *"A"*, *"B"*, etc.): it represents the beliefs held by an agent. Sets closed under logically consequence--those sets in which every sentence following logically from this set is already in the set--are called *Belief sets* (indicated by *"K" and "H"*). In other words, a belief set is a propositional formula and its logical consequences.[40]
  - *Consequence operator:* a function *Cn* from sets of sentences to sets of sentences, satisfying the following conditions:
    - *Inclusion:* $A \subseteq Cn(A)$.
    - *Monotonicity:* If $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$.
    - *Iteration:* $Cn(A) = Cn(Cn(A))$.
  - *Some properties of Cn:*
    - *Supraclassicality:* if a sentence p can be derived from a language A, which contains it, by classical truth-functional logic, then $p \in Cn(A)$.
    - *Deduction:* $q \in Cn(A \cup p)$ if and only if $p \rightarrow q \in Cn(A)$.
    - *Compactness:* if $p \in Cn(A)$ then $p \in Cn(A')$, where A' is a finite subset of A.

[40] Gärdenfors called a Belief set also "knowledge set" and he said that "[...] it is a partial description of the world. "Partial" because in general there are sentences ϕ such that neither ϕ nor ¬ϕ are in K". Peter Gärdenfors, "Belief revision: an introduction", in *Belief Revision*, ed. P. Gärdenfors (Cambridge: Cambridge University Press, 1992), 6.

If $A = Cn(A)$ then A is logically closed with respect to Cn and, as usual, it is a belief set.

- Epistemic attitudes (for a sentence p with respect to a belief set K):
  - *Acceptance:* p is accepted if and only if $p \in K$. In that case the agent believes p.
  - *Rejection:* p is rejected if and only if $\sim p \in K$. In that case the agent does not believe p.
  - *Indetermination:* p is undetermined if $p \notin K$ and $\sim p \notin K$. In that case the agent neither believes nor does not believes p.
- Types of belief change (for a belief set K):
  - *Expansion* (+): a new belief-representing sentence p is simply added to K regardless of preserving consistency. Its result is defined as $K+p = Cn(K \cup \{p\})$. Expansion is the simplest AGM belief change's operator and it satisfies this set of postulates:

    [K+1]. *Closure:* $K+p$ is a belief set, i.e. it is closed under logical consequence.

    [K+2]. *Success:* $p \in K+p$.

    [K+3]. *Inclusion:* $K \subseteq K+p$.

    [K+4]. *Vacuity:* if $p \in K$, then $K+p = K$, i.e. if p already belong to K, there is no expansion to do.

    [K+5]. *Monotonicity:* if $K \subseteq H$, then $K+p \subseteq H+p$, where H is a belief set including K.

    [K+6]. *Minimality:* $K+p$ is the smallest belief set satisfying [K+1]—[K+5]. Minimality can be considered as an expression of the principle of *minimal change of belief*, one of the main criterion of belief change in AGM. It consists in making the smallest possible change to receive the new information.[41]

[41] "[...] when we change our beliefs, we want to retain as much as possible from our old beliefs—we want to make a minimal change. Information is in general not gratuitous, and unnecessary losses of

- *Revision* ($*$): a new belief-representing sentence p is added to K preserving consistency. So, differently from expansion, revision should avoid inconsistency, eliminating that sentences which generate any contradiction with the new evidence. Therefore, revision is not an operation as simple as expansion and its result $K*p$ is defined with this set of postulates:

  [K∗1]. *Closure:* $K*p$ is a belief set.

  [K∗2]. *Success:* $p \in K*p$.

  [K∗3]. *Inclusion:* $K*p \subseteq K+p$.

  [K∗4]. *Vacuity:* if $\sim p \notin K$, then $K+p \subseteq K*p$, i.e. if $\sim p$ does not belong to K, there is no contradiction and no reason to remove anything.

  [K∗5]. *Consistency:* $\bot \in K*p$ if and only if $\vdash \sim p$, i.e. unless the new sentence is itself inconsistent, then the revised belief set is consistent.

  [K∗6]. *Extensionality:* if $p \equiv q$, then $K*p = K*q$, i.e. if two sentences are logically equivalent, their revision yields the same result.

  [K∗7]. *Superexpansion:* $K*(p \wedge q) \subseteq (K*p)+q$, i.e. a revision by conjunction can be made first revising K with respect to p and then expanding $K*p$ by q, on condition that q does not contradict $K*p$.

  [K∗8]. *Subexpansion:* if $\sim q \notin K*p$, then $(K*p)+q \subseteq K*(p \wedge q)$, i.e. for the same intuition captured by postulate [K∗7], if q does not contradict the revised belief set, then the expansion of $K*p$ by q is a subset of $K*(p \wedge q)$.

- *Contraction* ($\dot{-}$): a belief-representing sentence p is eliminated from K, without adding any new belief. Usually this kind of operator works when either some doubt exists about a belief either the agent intends temporally to suspend its belief on a sentence. Contraction postulates are:

information are therefore to be avoided. This heuristic criterion may be called the criterion of informational economy." Peter Gärdenfors, "Belief revision: an introduction", 9.

[K∸1]. *Closure:* $K*p$ is a belief set.

[K∸2]. *Success:* if $\nvdash p$, then p $\notin K \dot{-} p$, i.e. the only sentences that cannot be contracted are the tautologies.

[K∸3]. *Inclusion:* $K \dot{-} p \subseteq K$.

[K∸4]. *Vacuity:* if $p \notin K$, then $K \dot{-} p \subseteq K$, i.e. if p does not belong to K, contraction operator has to do nothing.

[K∸5]. *Recovery:* $K \subseteq (K \dot{-} p) + p$, i.e. for recovering the original belief set it is enough to add the removed sentence by expansion.

[K∸6]. *Extensionality:* if $p \equiv q$, then $(K \dot{-} p) = (K \dot{-} q)$, i.e. if two sentences are logically equivalent, their contraction yields the same result.

[K∸7]. *Conjunctive inclusion:* If $p \notin K \dot{-} (p \wedge q)$, then $K \dot{-} (p \wedge q) \subseteq K \dot{-} p$.

[K∸8]. *Conjunctive overlap:* $(K \dot{-} p) \cap (K \dot{-} q) \subseteq K \dot{-} (p \wedge q)$, i.e. those beliefs preserved in $(K \dot{-} p)$ and $(K \dot{-} q)$ are also maintained in $K \dot{-} (p \wedge q)$.

- Relation between revision and contraction:
  - *Harper Identity:* $K \dot{-} p = (K * \sim p) \cap K$, i.e. the result of eliminating p from K equals those beliefs that are retained after revising K by p. Harper Identity defines contraction in terms of revision, so that revision appears as a primitive operator.
  - *Levi Identity:* $K * p = (K \dot{-} \sim p) + p$, i.e. a revision of K by p can be made first contracting K by $\sim p$ and then simply expanding it by p. In this way contraction is presented as a primitive operator and revision is defined in terms of it.

Epistemic states associated with belief sets have been represented in several way. One of those is by *Grove orderings*, known for providing a semantic model for AGM theory, called *sphere semantics*[42]-- inspired by Lewis' semantics for counterfactuals--and for making a representation for AGM.

[42] Adam Grove, "Two Modelling for Theory Change", Journal of Philosophical Logic, 17 (1988): 157--170.

Gärdenfors developed an epistemic semantic model for conditionals based on AGM theory[43], so interpreting conditionals sentences in terms of belief revision.

The basic idea is that a sentence gets its meaning in correspondence, not with a world, but with a belief system. According to Gärdenfors, a belief system consists of:

"(1) a class of models of epistemic states, (2) a valuation function determining the epistemic attitude in the state for each epistemic state, (3) a class of epistemic inputs, and (4) an epistemic commitment function that for a given state of belief and a given epistemic input, determines a new state of belief."[44]

In this background the Ramsey's Test plays the crucial role of *acceptability principle* for sentences like "If *p* then *q*". So, Gärdenfors interpreted in a very naturally way that famous and tricky pass of Ramsey 1929 in terms of AGM revision, giving a suppositional interpretation of Ramsey's Test different from Adams:

- *Gärdenfors Ramsey Test* (GRT): $p \rightarrow q \in K$ if and only if $q \in K * p$, i.e. a conditional is accepted if and only if its consequent is contained into the belief set revised by the antecedent.

Unfortunately, Gärdenfors proved that GRT is not compatible with those AGM's postulates which equalize the classic *preservation condition*--[K∗4] and [K∗8]--and that it works just on pain of making AGM trivial.[45] So, Gärdenfors himself interpreted his result as a defeat of AGM account for conditionals.

In front of one more triviality result--also known as "Gärdenfors' Triviality Result"--several solution to avoid it have been advanced, included the "extreme" position to definitely reject the Ramsey's Test as acceptability criterion for conditionals.[46] However, other alternatives, for example that one proposed by Levi--

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[43] Peter Gärdenfors, *Knowledge in flux, modeling the dynamics of epistemic states*, (Cambridge MA: MIT Press, 1988).

[44] Horacio Arló Costa, Isaac Levi, "Two Notions of Epistemic Validity", Synthese, vol. 109 (1996), 222.

[45] Peter Gärdenfors, Sten Lindström, Michael Morreau, Wlodzimierz Rabinowicz, "The negative Ramsey test: Another triviality result", *The Logic of Theory Change, Lecture Notes in Computer Science*, Vol. 465 (1991): 127--134.

[46] Hans Rott, "Ifs, though and because", *Erkenntnis*, vol. 25 (1986): 345--370.

also shared by Arló Costa[47]--demonstrated that Gärdenfors' result could not be avoid simply denying the Ramsey Test.[48]

Levi accepted the Ramsey Test as acceptability principle for conditionals, but rejecting the idea that belief sets includes such conditional sentences as elements of it. He says:

"[…] a conditional of the (regimented) type h > g is a judgment concerning the serious possibility of g relative to a *transformation* of the current corpus or belief set K expressible in L and not relative to the current corpus itself. The transformation T(K) of K is the L-minimal revision of K which is subject to the sole constraint that h be a member of T(K). […] Consequently, conditional sentences ought not to be construed as truth-value-bearing any more than judgments of serious possibility ought to be. They are expressions of our evaluations of truth-value-bearing hypotheses with respect to serious possibility relative to transformations of the current corpus (Levi 1977, 1980)."[49]

Interpreting conditional sentences as serious possibilities about a belief set--not as members of the belief set itself--Levi makes that the preservation condition cannot affect them. According to him this is the only manner to avoid a trivialization, given that every revision postulates is restricted to a propositional belief set and to its elements. So, he considers GRT such as no the most appropriate formula for translating Ramsey's idea, because now we need a notion able to represent a principle of acceptability for sentences holding a cognitive content but lacking truth-values. Therefore, Levi proposed such a reformulation of GRT:

- *Levi Ramsey Test* (LRT): if $p, q \in L_0$ then $p \rightarrow q \in s(K)$ if and only if $q \in K * p$, whenever K is consistent--where L_0 is a Boolean language free of

[47] Horacio Arló Costa, Isaac Levi, "Two Notions of Epistemic Validity", 217--262.

[48] Isaac Levi, "Iteration of Conditionals and the Ramsey Test", Synthese, vol. 76 (1988): 49--81. Also Arlo-Costa shared the same position, see Horacio Arló Costa, Isaac Levi, "Two Notions of Epistemic Validity", Synthese, vol. 109 (1996), 217--262.

[49] Isaac Levi, "Iteration of Conditionals and the Ramsey Test", 61--62.

modal and epistemic operator and *s(K)* is a "support set" closed under logical consequences such that $s(K) \supseteq K$.[50]

Honestly, I find Levi's argument really interesting because, excluding conditional sentences from the belief set and speaking about them not in terms of truth but in terms of acceptability conditions, he takes a position *à la* Adams but in a belief revision background. His hard judgment against keeping on trying to conciliate Ramsey Test with a possible world semantics captured my attention too. He established that the origin of misunderstanding conditional sentences resides again in such attempt to analyze Stalnaker and Lewis' view in terms of Ramsey test for conditionals' acceptability--even if, now, by a belief-revision account:

"The moral of the story would seem to be that efforts to reconstruct a theory of conditionals along the lines of Stalnaker and Lewis in terms of belief revisions ought to be abandoned. Such theories cannot be reconstructed along such lines. If they make sense at all, they make sense within a framework which takes realism about possible worlds seriously. I for one cannot find it in my heart to embrace such metaphysics gratuitously. Gärdenfors exhibits a similar penchant but, at the same time, displays a devotion to the Stalnaker-Lewis ideas. The need to accommodate Stalnaker-Lewis is so great that he seems prepared to give up the core of the belief revision approach - to wit, that bodies of knowledge define the spaces of serious possibility."[51]

However, also Levi's account presents several problems, like those ones usually connected with denying that conditionals are true or false. In add, even his criticism of Gärdenfors' truth semantics does not seem to work really. Indeed, although the problems of Gärdenfors' account are unquestionable, it definitely appears to survive to Levi's criticism.[52] On the other hand, we should admit that Levi's arguments deserve the merit to have induced us to pay more attention to AGM's formalism.

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[50] Horacio Arló Costa, Isaac Levi, "Two Notions of Epistemic Validity", 225--226.

[51] Isaac Levi, "Iteration of Conditionals and the Ramsey Test", 69.

[52] About this point, see Roger. D. Rosenkrantz, "Review: For the Sake of the Argument. Ramsey Test Conditionals, Inductive Inference, and Nonmonotic Reasoning by Isaac Levi", *The Journal of Symbolic Logic*, vol. 62, No. 3 (1997): 1041--1043.

In conclusion, in a belief revision's context both the attempt to conciliate Ramsey Test with the semantics of possible worlds and the extreme position to deny truth conditions for conditionals preserve many troubles.

II

STALNAKER'S ANALYSIS

1. A semantic for conditional statements

It is now the moment to talk about the great contribute provided by Robert Stalnaker--in collaboration with Richmond Thomason--so that any reader can easily understand why several philosophers holding a suppositional account for conditionals, like Gärdenfors et al., really wished to conciliate their view with Stalnaker's non-truth-functional account.

When Stalnaker developed his theory, in 1968, he had in mind a semantics able to provide truth conditions for conditionals--first for counterfactuals but then extending it to indicatives too[53]--in accordance with Ramsey-Adams Thesis. Indeed, in front of quite unsatisfactory theories--like the material implication analysis --Stalnaker thought to consider Ramsey's Test, even though making some adjustments--or at least trying to generalize it, given that Ramsey spoke only about situations where the agent has no idea about the antecedent's truth-value. So, according to Stalnaker, this is the procedure for deciding whether (or not) believe to a conditional statement:

"First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustment are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true."[54]

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[53] "The analysis was constructed primarily to account for counterfactual conditionals – conditionals whose antecedents are assumed by the speaker to be false – but the analysis was intended to fit conditional sentences generally, without regard to the attitudes taken by the speaker to antecedent or consequent or his purpose in uttering them, and without regard to grammatical mood in which the conditional is expressed." Robert C. Stalnaker (1976), "Indicative conditionals", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 198.

[54] Robert C. Stalnaker (1968), "A Theory of Conditionals", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 41--55.

Once established belief-conditions for conditionals, how to fix truth-conditions? In this regard Stalnaker introduced the concept of *possible world*, which represents the "ontological analogue of a stock of hypothetical beliefs"[55], so that conditionals' truth-conditions for conditionals can be provided by an adaptation of truth-conditions settled by the possible world semantics:

"Consider a possible world in which *A* is true, and which otherwise differs minimally from the actual world. *"If A, then B" is true (false) just in case B is true (false) in that possible world*."[56]

Applying for an analysis in terms of possible worlds, Stalnaker built a semantical system (C2) for conditionals using Kripke's semantics for modal logic:[57]

- Formal system:
  - *Conditional formulas:* every expressions of the form "$p > q$". If p and q are well-formed formulas (wff), then also $p > q$ is a wff. Other kinds of connectives ($\wedge$, $\vee$, $\sim$, $\equiv$, $\supset$) are defined as usual. Biconditional connective ($\gtreqless$) can be defined as:
    - $p \gtreqless q = (p > q) \wedge (q > p)$.
  - *Modal formulas:* every expression like "$\Box p$" and "$\Diamond p$", defined as:
    - $\Box p = \sim p > p$.
    - $\Diamond p = \sim(p > \sim p)$.
  - Rules of inference:
    - *Modus Ponens:* $p > q$, $p \vdash q$, i.e. if p and $p > q$ are theorems, then q is a theorem.
    - *Modus Tollens:* $p > q$, $\sim p \vdash \sim q$, i.e. if $\sim p$ and $p > q$ are theorems, then $\sim q$ is a theorem.

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[55] Robert C. Stalnaker, "A Theory of Conditionals", 45.

[56] Robert C. Stalnaker, "A Theory of Conditionals", 45.

[57] The whole system could be found in Robert C. Stalnaker and Richmond H. Thomason, "A semantic analysis of conditional logic", *Theoria*, vol. 36 (1970): 23--42, and in Robert C. Stalnaker, "A Theory of Conditionals", 41--55.

- *Gödel rule of necessitation:* if p is a theorem, then also $\Box p$ is a theorem.
  - Axioms:

    [A1]. Any tautologous wff is an axiom.

    [A2]. $\Box(p \supset q) \supset (\Box p \supset \Box q)$.

    [A3]. $\Box(p \supset q) \supset (p > q)$.

    [A4]. $\lozenge p \supset [(p > q) \supset \sim(p > \sim q)]$.

    [A5]. $p > (q \vee z) \supset [(p > q) \vee (p > z)]$.

    [A6]. $(p > q) \supset (p \supset q)$.

    [A7]. $p \gtreqless q \supset [(p > z) \supset (q > z)]$.

    Axioms [A3] and [A6] make ">" a kind of intermediate connective between strict implication and material conditional, despite keeping some difference with respect to the other ones. Indeed, the following properties--valid for strict implication or material conditional or for both --do not hold in C2:[58]

    - *False antecedent:* $\sim p \vdash p > q$.
    - *True consequent:* $q \vdash p > q$.
    - *Material negation:* $\sim(p > q) \vdash p$.
    - *Simplification of disjunctive antecedents:* $(p \vee q) > z \vdash (p > z) \wedge (q > z)$.
    - *Antecedent preservation:* $\vdash p > (q > p)$.
    - *Import-Export:* $p \rightarrow (q \rightarrow z) \vdash (p \wedge q) \rightarrow z$.
    - *Transitivity:* $p > q,\ q > z \vdash p > z$--">" cannot be iterated.
    - *Contraposition:* $p > q \vdash \sim q > \sim p$.
    - *Antecedent Strengthening:* $p > q \vdash (p \wedge z) > q$.
    - $\sim(p > q) \equiv (p > \sim q)$, given $\lozenge p$.

- Semantics:
  - *Model structure (M):* a triple structure *⟨K, R, λ⟩. K* represents the set of possible world. *R* is the relation of relative possibility (or relation of

[58] William B. Starr, "Indicative Conditionals, Strictly", Cornell University (2014): 9--10.

accessibility) defining the model structure. In other words, R is the relation between worlds and wRw' means "w is possible with respect to w' ". λ is the only element of M not concerning Kripke's semantics and it represents the absurd world--a world of K in which all formulas, even contradictions and their consequences, are true. Stalnaker introduced it for interpreting those conditionals whose antecedent is impossible.

- *Selection-function (f):* a function taking a proposition and a possible world as its arguments and a possible world as its value. So, given a selection-function $f(p, w) = w'$, p is the antecedent of a conditional, w is the base world, and w' is the selected world. These are the conditions that a selection-function has to observe:
  - For all antecedent p and every base world w, p must be true in $f(p, w)$, i.e. the antecedent must be true in the selected world, so that w' is also called p-world.
  - For all antecedent p and every base world w, $f(p, w) = \lambda$ when it is not possible any world with respect to w where p is true, i.e. the selection-function is an absurd world just when the antecedent is impossible, so that there are no p-worlds.
  - $f(p, w)$ has to select the *closest* p-world to w, i.e. the selection-function must take, if possible, the most *similar* possible world to the base world.
  - For all antecedent p and every base world w, if p is true in w, then $f(p, w) = w$, i.e. if the base world w is a world in which the antecedent is true, then it will be selected.
  - For all antecedent p and p' and every base world w, if p is true in $f(p', w)$ and p' is true in $f(p, w)$, then $f(p, w) = f(p', w)$.
- *Semantical rules* for $p > q$:
  - $p > q$ is true in w if q is true in $f(p, w)$, i.e. a conditional turns out to be true in the base world when its consequent is true in the selected world.

- $p > q$ is false in w if q is false in $f(p, w)$, i.e. a conditional is false in the base world when its consequent is false in the selected world.
  - *Limit assumption:* for all antecedent p and every base world w there is *at least* one p-world--a world where p is true--different from w.
  - *Uniqueness assumption:* for all antecedent p and every base world w there is *just one* closest p-world, i.e. there never are two equally accessible worlds to w where p is true.
  - *Conditional excluded middle* (CEM): $(p > q) \vee (p > \sim q)$.
- Pragmatic restrictions (at least for indicative conditionals):
  - If $w \in C$, then $f(p, w) \in C$, i.e. for every world w in the context set C, the closest p-world must, pertain, if possible, to the same context set too. A *context set* is the set of worlds *epistemically* possible for the agent. In other words, since a lot of possibilities are usually taken for granted in a conversion, a context set is that set of worlds compatible with those possibilities. Stalnaker specified that a selection-function cannot be defined in terms of C. However, a context set can help in ordering possible worlds: every world in C is closer to w than any other one outside it.

Mentioning pragmatic restrictions, Stalnaker held a strict relation between semantic and pragmatic--nowadays no longer questioned--, claiming that the ambiguity of a conditional statement does not pertain to the semantic level, but to the pragmatic one. In other words, a conditional is not semantically ambiguous, but just pragmatically. Though their pragmatical ambiguity, conditionals own a common structure which gives them a single meaning and reveals their truth conditions. That common structure is given just by a semantics for conditional logic.

Invoking the link between semantic and pragmatic, Stalnaker tried also to explain the problem related with the *direct argument*--$p \vee q \vdash \sim p > q$.[59] Indeed, such a schema is invalid for Stalnaker's account because if we admit the validity of the

[59] Robert C. Stalnaker (1976), "Indicative conditionals", 193--194.

direct argument then "$>$" is logically equivalent with the material conditional "$\supset$", and we should also accept several absurd inferences. For example, let us consider the following direct argument:

[1] "Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did".

The validity of [1] makes "$>$" logically equivalent to the material conditional, so we should also assume any paradoxical cases related to "$\supset$", like this:

[2] "The butler did it. Therefore, if the butler didn't do it, the gardener did".

But, while [1] seems intuitively valid, [2] does not at all. On the other hand, both sentences have the same conclusion, and the premise of [2] entails the premise of [1]. Consequently, considering semantically valid the direct argument we are assuming "$\supset$" such as indicative conditional. And, rejecting the $\supset$-analysis, we have to reject the direct argument too.

Now, Stalnaker's solution consists in considering *semantically invalid* both [1] and [2], because the premise of [1] does not semantically entail its conclusion. Assuming the validity of the direct argument represents a mistake due to those rules holding in every conversation. Hence, Stalnaker suggested--like Grice did, but with different purpose--to look, not only at the semantic content of a sentence, but also at the pragmatic principles governing any discourse.

2. Stalnaker's probability system

A defense of Stalnaker's theory is given by drawing a connection between his semantics and the theory of conditional probability--that in those years was getting well received. So, Stalnaker built a probability system C2 by three steps, and each step represents a probability system, extension of the previous one:[60]

[60] That argument could be found in Robert C. Stalnaker (1970), "Probability and conditionals", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 107--128.

- First system (P1): it combines a truth valuation function v and an absolute probability function Pr compatible each other, showing that they are not exclusive alternatives, but complementary ones.
  - *P1-interpretation:* an order $\langle v, Pr \rangle$ where, for every proposition p, if $Pr(p)$=1, then $v(p)$=1. P1 represents a possible world and the state of knowledge an agent has in that world.
  - *Truth valuation function* (v): any function v, whose value is into {1; 0}, representing a possible world. Given two propositions p and q, v has to meet these conditions:
    - $v(p)$=1 if p is true in the world represented by the truth valuation function, otherwise $v(p)$=0.
    - $v(\sim p) = 1 - v(p)$.
    - $v(p \wedge q) = v(p) \cdot v(q)$
  - *Absolute probability function* (Pr): any function Pr representing a *state of knowledge*. It assigns value 1 for those propositions known to be true, and value 0 for those ones known to be false. In addition, it includes a measure between {1; 0} for the degree to which an agent has right to believe propositions known neither to be true nor to be false. Given two proposition p and q, these are the conditions Pr has to meet:
    - $1 \geq Pr(p) \geq 0$.
    - $Pr(p) = Pr(p \wedge p)$.
    - $Pr(p \wedge q) = Pr(q \wedge p)$.
    - $Pr(p \wedge (q \wedge z)) = Pr((q \wedge p) \wedge z)$.
    - $Pr(p) + Pr(\sim p) = 1$.
    - $Pr(p) = Pr(p \wedge q) + Pr(p \wedge \sim q)$.

    $Pr(p)$ could be interpreted in terms of bet, where Pr is a number r determining the minimum odds an agent would be willing to accept in a bet on the truth of p. Given r, the agent should be willing to bet on p at odds no less favorable than $r/(r-1)$, i.e. the bet should not be less than the ratio between the probability the proposition is true (= the bet is won) and the probability it is false (= the bet is lost). If there is no set of bets such that the agent is sure to loose, then the *Pr* is *coherent*. In addition, a coherent probability function is called *strictly coherent* when there is no set of bets such that the agent might loose and cannot possibly win.

- *Class of epistemically possible worlds* (K): given a state of knowledge *Pr*, *K* is a class of possible worlds compatible with *Pr*, defined as:
  - $K = \{v / \langle v, Pr \rangle$ is a P1-interpretation$\}$.

  If $Pr(p) = 1$ then *p* is true in *K*, i.e. it is true in every possible epistemic world. In that case P1 is strictly coherent. So, the requirement a coherent *Pr* has to meet to be strictly coherent is:
  - If $Pr(p)=1$ then *p* is true in all possible outcomes.[61]
- *Conditional probability* ($Pr(q, p)$): it represents reasonable odds for a conditional bet.[62] Given two events *p* and *q*, where their conditional probability is a number *r*, the agent should be willing to bet that *q* on the condition *p* at odds of $r/(r-1)$. So, it is appropriate to interpret $Pr(q, p)$ in terms of absolute probability:
  - $Pr(q,p) = \frac{Pr(p \wedge q)}{Pr(p)}$, where $Pr(p) \neq 0$.

  When $Pr(p)=0$, $Pr(q, p)$ is undefined.

  A Conditional probability is just a ratio between two absolute probabilities. However, because [P1] does not cover counterfactual probabilities, an extension of that is needed.

- Second system (P2): as extension of P1, it combines a truth valuation function *v* and a probability function *Pr* as well. But in this second system *Pr*

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[61] Also known as "Kemeny's condition" because presented by Kemeny in: John G. Kemeny, "Fair bets and inductive probabilities", *The Journal of Symbolic Logic*, vol. 20 (1955): 263--273. It should be pointed out that this condition is not generally satisfied whether the set of possible outcomes is an infinite set. But, of course, that is not a case of Stalnaker's analysis.

[62] See note 18.

is a conditional probability function--which is not primitive, but easily defined as $Pr(p) = Pr(p, \mathrm{T})$, that is a special case of conditional probability.

- *P2-interpretation:* an order $\langle v, Pr \rangle$ where, for every proposition p and q, if $Pr(q, p) = 1$, then $v(p \supset q) = 1$.
- *Extended probability function*: any conditional probability function meeting these conditions:
  - $Pr(q, p) \geq 0$.
  - $Pr(q, q) = 1$.
  - If $Pr(q, p) = Pr(p, q) = 1$, then $Pr(z, q) = Pr(z, p)$.
  - $Pr(q \wedge p, z) = Pr(p \wedge q, z)$.
  - $Pr(q \wedge p, z) = Pr(q, z) \cdot Pr(p, q \wedge z)$.
  - If $Pr(\sim z, z) \neq 1$, then $Pr(\sim p, z) = 1 - Pr(q, z)$.

  An extended probability function represents not only an extended state of knowledge, but also a set of *hypothetical* state of knowledge, since it measures the degree to which an agent has a right to believe a proposition q but also to which it would have a right to believe q if he knew some condition p--but that instead it does not know. By the way, if the condition is known to be true then $Pr(p, \mathrm{T}) = 1$, so that $Pr(q, p) = Pr(q, \mathrm{T})$. When q is a tautology, conditional knowledge is a *tout court* knowledge.
- *Impossible proposition:* a proposition whose negation is known to be true, i.e. when $Pr(p, \sim p) = 1$.
- *Absurd state of knowledge:* a state of knowledge assuming an impossible proposition as true.
- *Extended belief function:* any conditional probability function that is an extended probability function too.

- Third system (P3): extension of P2 by introducing conditional propositions and making a change in the object language. His system represents the whole C2 system.

- *C2 Object language:* it is the same previously language, with the exception that the connective “>” is now a primitive symbol and, if p and q are wffs, then also $p > q$ is a wffs.
- *C2 extended probability function:* a function meeting all of the conditions of an ordinary extended probability function.
- *Absolute probability of a conditional proposition* ($Pr(p > q)$): it must be equal to the conditional probability of the consequent q on the condition of the antecedent p:
  - $Pr(p > q) = Pr(q, p)$.
- *Probabilistic simultaneously satisfaction:* a class of proposition is p-simultaneously satisfiable when every member might be known to be true.
- *Probabilistic validity:* a wff is p-valid if its negation is not p-simultaneously satisfiable, i.e. whose negation cannot be known to be true.
- *Definitions of modal operators:*
  - $\Box p = \sim p > p$.
  - $\Diamond p = \sim \Box \sim p$.
- *C2 Rules:*
  - If $p \supset q$ and p are theorems, then q is a theorem.
  - If p is a theorem, then $\Box p$ is a theorem.
- *C2 Axioms:*
  - Any tautologous wff is an axiom.
  - $\Box(p \supset q) \supset (\Box p \supset \Box q)$.
  - $\Box(p \supset q) \supset (p > q)$.
  - $\Diamond p \supset [(p > q) \supset \sim(p \supset \sim q)]$.
  - $[p > (q \vee z)] \supset [(p > q) \vee (p > z)]$.
  - $(p > q) \supset (p \supset q)$.
  - $\{[(p > q) \wedge (q > p)] \supset (p > z)\} \supset (q > z)$.
- *Object language theorems:*

- $\vdash (T > p) \equiv p.$
- $\vdash p > p.$
- $\vdash \Diamond z \supset [(z > p) \equiv \sim(z > \sim p)].$
- $\vdash z > (p \wedge q) \equiv [z > (q \wedge p)].$
- $\vdash z > (p \wedge q) \equiv \{[(z > p) \wedge [(p \wedge z) > q]\}.$

- *Semantical completeness theorem:* a class of wffs of C2 is p-simultaneously satisfiable iff it is C2-consistent.
- *Derived rules and theorems:*
  - If $\vdash p$, then $\vdash z_1 > [z_2 > \ldots > (z_n > p)].$
  - If $\vdash p \supset q$, then $\vdash (z > \text{p}) \supset (z > q).$
  - $\vdash z > (p \equiv q) \equiv [(z > p) \equiv (p > q)].$
  - $\vdash z > (p \wedge q) \equiv (z > p) \wedge (z > q).$

In conclusion, Stalnaker developed a parallelism between his semantics and the theory of conditional probability, showing that the theorems of C2 are nevertheless the valid sentences of Ramsey's Test.

3. Lewis revision of Stalnaker's account

It is well known that Stalnaker's theory of conditionals is really close to David Lewis' analysis, so that we may easily find several texts speaking about Stalnaker-Lewis' approach. In spite of that, their accounts conserves some differences, other than the fact that Lewis' theory interests only counterfactual conditionals-- symbolized by "□→".

First of all, Lewis rejected the *Uniqueness assumption* considering it unjustified. He simply showed how hard is choosing one closest *p*-world, leading to think that there might be more equally closest *p*-worlds.[63] Consequently, Lewis rejected the

[63] "Example: *A* is 'Bizet and Verdi are compatriots', *F* is 'Bizet and Verdi are French', *I* is 'Bizet and Verdi are Italian'. Grant for the sake of the argument that we have the closest *F*-world and the closest *I*-world; that these are distinct (dual citizenships would be a gratuitous difference from actuality); and that these are the two finalists in the competition for closest *A*-world''. David Lewis, "Counterfactual and comparative Possibility", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*,

law of *Conditional Excluded Middle* because, if we do not assume the *Uniqueness assumption*--simply thinking that there might be more equally closest *p*-worlds--, why should we accept CEM? Even assuming it works, it would allow us to choose just one single world. In addition, it really does not work! Indeed, although the objection could seem *prima facie* inconsistent--according to any ordinary language--there could be found several counterexamples showing CEM does not work.[64] So, even though it could be plausible in support of the thesis $\sim(p \Box\!\!\rightarrow q) \equiv p \Box\!\!\rightarrow \sim q$, it could not be considered at all such as a general criterion for similarity between worlds in which the antecedent *p* is true.

Another objection Lewis raised against Stalnaker's theory is that it loses the difference between "would" and "might". Indeed, Lewis proposed a definition of *might*-conditional in terms of *would*-conditional, so that a *might*-conditional "$\Diamond\!\!\rightarrow$" is defined in such a way: $p \Diamond\!\!\rightarrow q = \sim(p \Box\!\!\rightarrow \sim q)$--where $p \Diamond\!\!\rightarrow q$ represents $p \Box\!\!\rightarrow \Diamond q$. Lewis' definition, together with CEM, implies that there is no difference in truth-values between "would" and "might", causing that $p \Diamond\!\!\rightarrow q$ implies $p \Box\!\!\rightarrow q$ and *vice versa* in both Stalnaker and Lewis' accounts--except in vacuous cases. But "this is obviously an inacceptable conclusion" for Stalnaker, as himself admitted[65], reason for what he could not define a might conditional such as Lewis did. On the other hand, Lewis pointed out that he could not find other manner to define "might" according to Stalnaker's account:

"How else could he define it? Four candidates come to mind: $\Diamond(\phi \,\&\, \psi)$, $\Diamond(\phi \Box\!\!\rightarrow \psi)$, $\phi \Box\!\!\rightarrow \Diamond \psi)$, and $\phi \Box\!\!\rightarrow \Diamond(\phi \,\&\, \psi)$. But none will do. Take ϕ as *'I looked in my pocket'* and ψ as *'I found a penny'*; suppose I didn't look, suppose there was no

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eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 60.

[64] The most common counterexample is: "It is not that case that if Bizet and Verdi were compatriots, Bizet would be Italian; and it is not the case that if Bizet and Verdi were compatriots, Bizet would not be Italian; nevertheless, if Bizet and Verdi were compatriots, Bizet either would or would not be Italian." David Lewis, *Counterfactuals* (Oxford: Blackwell, 1973), 80.

[65] Robert C. Stalnaker, "A Defense of Conditional Excluded Middle", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 98.

penny to be found, and make commonplace assumptions about relevant matters of fact. Then *'If I looked, I might have found a penny'* is plainly false, but all four candidate symbolization are true. $\phi \& \psi$ is false, but only contingently so; hence $\Diamond(\phi \& \psi)$ is true. $\phi \Box\!\!\rightarrow \psi$ is false, but again only contingently so; hence $\Diamond(\phi \Box\!\!\rightarrow \psi)$ is true. If I had looked, ψ and $(\phi \& \psi)$ would have been false, but again only contingently so; hence $\phi \Box\!\!\rightarrow \Diamond \psi$ and $\phi \Box\!\!\rightarrow \Diamond(\phi \& \psi)$ are true. [...]"[66]

Lewis disapproves also the *Limit assumption* because, even not assuming exactly one closest *p*-world, it seems to suggest proceeding to closer and closer *p*-worlds until get to an end. The *Limit assumption* completely excludes the possibility to proceed infinitely, and this is, once again, unjustified[67].

In conclusion, Lewis proposed such a revision of Stalnaker's theory: to select a *set* of possible *p*-worlds that will equal the original Stalnaker's selection-function in case the set contains a single world, but it will not if, and it isn't out of the question, the set contains more--finitely or infinitely--*p*-worlds. So, a counterfactual $p \Box\!\!\rightarrow q$ is true in the actual world if and only if *some* accessible *p*-world in which q is true is closer to the actual one than any *p*-world in which q is false. In the same way, $p \Diamond\!\!\rightarrow q$ is true in the actual world if and only if there are accessible *p*-worlds and, for every accessible *p*-world in which q is false, there is some *p*-world in which q is true that is at least as close to the actual world as it is. Now, in such a revision of Stalnaker's account we need assumptions other than those of Uniqueness and Limit. According to Lewis we should assume:[68]

[66] David Lewis, *Counterfactuals* (Oxford: Blackwell, 1973), 80--81.

[67] "Example: *A* is 'I am over 7 feet tall'. If there are closest *A*-worlds to ours, pick one of them: how tall am I there? I must be 7+ε feet tall, for some positive ε, else it would not be an *A*-world. But there are *A*-worlds where I am only 7+ε/2 feet tall. Since that is closest to my actual height, why isn't one of these worlds closer to ours than the purportedly closest *A*-worlds where I am only 7+ε feet tall? And why isn't a suitable world where I am only 7+ε/4 feet even closer to ours, and so ad infinitum? (In special cases, but not in general, there may be a good reason why not. Perhaps 7+ε could have been produced by a difference in one gene, whereas any height below that but still above 7 would have taken differences in many genes). If here are *A*-worlds closer and closer to *i* without end, then any consequent you like holds at every closest *A*-world to *i*, because there aren't any. If I were over 7 feet tall I would bump my head on the sky". David Lewis, "Counterfactual and comparative Possibility", 63.

[68] David Lewis, "Counterfactual and comparative Possibility", 63--64.

- An *Ordering assumption:* for every world *w*, a similarity relation produces a *weak ordering* of those worlds accessible to w, such that $w' \leqslant_w w''$ means "*w''* is not closer to *w* than *w'*"--where $\leqslant_w$ is *connected* and *transitive*.
- A *Centering assumption:* every world is accessible and closer to itself than any other world.

I want to point out that Lewis, like Stalnaker, thought the closeness between worlds in terms of *similarity*.[69] But he tried, according to me, to give us a little more sophisticated definition[70], identifying the similarity order in relation to the natural laws governing every world. So, the "best system" is that set of worlds totally equal to the actual one with reference to the natural laws, but with the only difference that a small "miracle", happened in a time *t*, made the antecedent true. Consequently, any particular fact *before t* is preserved, but some other facts *after t* are not.[71]

However, Lewis' revision of Stalnaker's theory does not solve completely those problems which it was previously designed for. Indeed, Stalnaker replied[72] that, first of all, a reformulation of his own theory in terms of $\leqslant_w$ represents just a special case of Lewis' account and, second, the assumptions of Uniqueness and Limit are not so simple to avoid because they denote an entailment principle in the semantics for conditionals. On the other hand, Stalnaker admitted that many assumptions made in an abstract semantic theory are not so well defined at the moment of their

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[69] Actually, in a personal conversation Stalnaker told me that he *never* gave a definition of "similarity". According to him, it is Lewis who talked explicitly about such a notion. Stalnaker limited to say that *the world which differs minimally from the actual one is the most similar*. However, although he did not logically define such a "similarity", this notion is certainly invoked. Therefore, even if I understand that "similarity" is an elusive concept, I think it is not an advantage--but rather a limit--to not have given a properly definition.

[70] However he cannot solve every problem that a so much ambiguous notion, like that of similarity, yield.

[71] David Lewis, *Counterfactuals*, 72--77.

[72] Robert C. Stalnaker, "A Defense of Conditional Excluded Middle", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 87--104.

application. Reason for what he proposed a general criterion of vagueness, identified with the *Theory of Supervaluations* developed by Van Fraassen[73].

A supervaluation consists in a two-stage valuation, after that both standard three-valued and two-valued valuation are defined. In the first stage, every formula ϕ is evaluated by a three-valued valuation. In the second stage, supervaluations are associated to the valuations, so that every ϕ is supervaluated. Given a two-valued valuation v_2, a three-valued valuation v_3 and a supervaluation s, s is so defined, for every ϕ:

- $s(\phi)$=t iff $v_2(\phi)=v_3(\phi)$=t.
- $s(\phi)$=f iff $v_2(\phi)=v_3(\phi)$=f.
- $s(\phi)$=n iff $v_2(\phi)\neq v_3(\phi)$.

In other words, for every formula ϕ, if every two-valued valuation coincides with every three-valued valuation about the truth-value assignment, then a supervaluation is associated to them, such that $v_2(\phi)=v_3(\phi)=s(\phi)$. So, basically, Van Fraassen proposed a partial semantic isomorphic to the truth-functional semantics of Kleene's three-valued logic[74], according to which every partially defined semantic interpretation--assigning truth-values by a two-value *classical valuation*--will be completed arbitrarily by the correspondent class of completed defined interpretations. Hence, a supervaluation determines that if and only if a sentence is true in *all* corresponding classical valuations then it is true and if and only if the sentence is false in *all* of them then it is false. But, when it is true in some classical valuations and false in other ones, the sentence is *neither true nor false*, i.e. it is a *truth-gap*.

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[73] Bas Van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic", *Journal of Philosophy* 68 (1966): 481--495, and Bas Van Fraassen, "Hidden Variables in Conditional Logic", *Theoria*, vol. 40 (1974): 176--190.

[74] Stephen C. Kleene, "On notation for ordinal numbers", *The Journal of Symbolic Logic*, Vol. 3, No. 4 (1938): 150--155. Kleene's three-valued logic is basically the same proposed by de Finetti (1935), but without mentioning the connective "|".

Recurring to Van Fraassen's theory, Stalnaker provided an argument in support of CEM showing that, because of the vagueness, there might be cases in which neither $p>q$ nor $p>\sim q$ are true. But, if a partial interpretation assumes a disjunct as true, its opposite cannot be assumed as well. So, not only CEM is safe, but the Uniqueness assumption too--contrary to what Lewis held.

About the argument against CEM involving the *might*-conditional, Stalnaker held that it does not work seriously. Indeed, it presupposes that Lewis' definition of "◊→" would be accepted. At that purpose, Stalnaker considered it too much simplistic to explain a complex structure like that of "might", while the right thing to do should be, first of all, inquiring it *outside* a conditional context and, then, in relation to a conditional analysis. Stalnaker suggested to consider a "might" occurring in a conditional context such as a standard "might": both a *might*-non-conditional and a *might*-conditional may express either an *epistemic possibility* either a *non-epistemic possibility*. However, given that most of *might*-conditionals manifest an epistemic possibility--whose scope is the whole conditional, not just the consequent--is unacceptable to conjoin it with the negation of the correspondent *would*-conditional--which expresses a necessity on the consequent--, as Lewis' interpretation allows. It is unacceptable, not because those conditionals are contradictories, but because their conjunction would be Moore-paradoxical.[75]

As a defense for the Limit assumption, Stalnaker invoked the notion of "relevance": the worlds have to be similar about *relevant respects*. Therefore, Lewis' counterexamples appear no appropriates since they show differences between worlds centered on irrelevant aspects--basically they are the same world--, so that the selection-function would not be possible.

In conclusion, Lewis' revision, rather than solve those problems of Stalnaker's account, seems to generate other complications, showing that a semantics of possible worlds, although really useful in analyzing conditional statements, still

[75] Moore's paradox: assertion like "*p*, but I don't believe *p*" and "*p*, but I believe that not-*p*". See Thomas Baldwin, *G. E. Moore, The Arguments of the Philosophers* (Routledge: London and New York, 1990), 226--232.

preserves some problems. However, Stalnaker specified that his analysis wants just to present the form of truth conditions of conditionals, not to discourage anybody to keep on studying such sentences.[76] He knew the problem was no solved but his theory represents definitely an important support. Reason for what several philosophers did not want to renounce, at least in a first moment, to an interpretation of conditional statement in terms of semantics of possible worlds.

Unfortunately, the question got more complicated when a proof involving compounds of conditionals, known as *Lewis' Triviality Result*, showed the incompatibility between Stalnaker's system and Ramsey's Test. Indeed, although C2 works in accordance with Ramsey's Test concerning simple conditionals, it fails with compound ones. Therefore, those supporters of both C2 and Ramsey's test had to make a not easy decision between a system representing one of the best proposal for a logic of conditionals and a well-recognized fundamental result in a decision theory. For this reason, a lot of philosophers, rather than opt for a choice, prefer proposing some solution to avoid the Triviality Result, in view to conciliate Stalnaker's theory and Ramsey's Test.

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[76] "It may seem that little has been accomplished by this analysis, since it just exchanges the problem of analyzing the conditional for the problem of analyzing a semantic function which is equally problematic, if not more so. In one sense this is correct: the analysis is not intended as a redaction of the conditional to something more familiar or less problematic, and it should not satisfy one who comes to the problem of analyzing conditionals with the epistemological scruples of a Hume or a Goodman. The aim of the analysis is to give a perspicuous representation of the formal structure of conditionals – to give the form of their truth conditions. Even if nothing substantive is said about how antecedents select counter factual possible worlds, the analysis still has non-trivial and in some cases surprising, consequences for the logic of conditionals." Robert C. Stalnaker, "Indicative conditionals", 198--199.

"[…] but a formal semantic analysis, by itself, is intended as neither a solution nor a dismissal of the problem of counterfactual conditionals. What such analysis purports to do is to clarify the abstract structure of a problematic concept in order to help separate formal problems about its logic form substantive problems." Robert C Stalnaker, *Inquiry*, (Cambridge MA: Bradford Books, MIT Press, 1984), 122.

III

LEWIS' TRIVIALITY RESULT AND ITS CONSEQUENCES

1. The Triviality Result

In 1976 Lewis presented an argument, known as *Triviality Result*, showing the incompatibility between the assumption that the probability of a proposition is *the probability it is true* and the *conditional probability*.[77] In such a way the divorce between Stalnaker's theory and the Equation is definitely formalized.

There are a lot of version of the Triviality Result, but I prefer reporting here the Lewis' original one[78]:

- Preliminaries:
  - Suppose we have a formal language containing at least the truth-functional connectives plus "→". Every connective could be used to compound any sentences in this language, whose truth-value is given in terms of possible worlds.
  - Define the conditional probability function in such a way:
    - $P(q \mid p) = P(q \wedge p) \mid P(p)$, if $P(p) > 0$[79].
  - Assume the following standard probability laws:
    - $1 \geq P(p) \geq 0$.
    - If p and q are equivalent--both true at the same world--, then $P(p) = P(q)$.
    - If p and q are incompatible--both true at no world--, then

[77] David Lewis, "Probabilities of Conditionals and Conditional Probabilities", in *Ifs: Conditionals, Belief, Decision, Chance, and Time*, eds. W. L. Harper, R. Stalnaker and G. Pearce (Dordrecht & Boston: Reidel Publishing Company, 1981), 129--147.

[78] A simpler version is that by Blackburn. See Simon Blackburn, "How Can We Tell Whether a Commitment has a Truth Condition?", in *Meaning and Interpretation*, ed. C. Travis (Oxford: Blackwell, 1986), 201--232.

[79] If $P(p) = 0$ then $P(q \mid p)$ remains undefined. A truthful speaker considers permissible to assert the indicative conditional p →q just in case $P(q \mid p)$ is sufficiently close to 1, i.e. only if $P(q \wedge p)$ is sufficiently greater that $P(\sim q \wedge p)$. David Lewis, "Probabilities of Conditionals and Conditional Probabilities", 129.

$P(p \vee q) = P(p) + P(q).$

    - If *p* is necessary--true in every worlds--, then $P(p)=1$.
  - Suppose to interpret "→" such that:
    - $P(p \rightarrow q) = P(q \mid p)$, if $P(p) > 0$, i.e. the probability of a conditional is its conditional probability.

  so that, if it holds, this holds too:

    - $P(p \rightarrow q \mid z) = P(q \mid p \wedge z)$, if $P(p \wedge z) > 0$.
- First Triviality Result:
  - Take $P(p \wedge q)$ and $P(p \wedge {\sim}q)$ both positive, so that $P(p)$, $P(q)$ and $P({\sim}q)$ are positive too. Now we have:
    - $P(p \rightarrow q) = P(q \mid p)$ holds by $P(p \rightarrow q) = P(q \mid p)$.
    - $P(p \rightarrow q \mid q) = P(q \mid p \wedge q) = 1$ and $P(p \rightarrow q \mid {\sim}q) = P(q \mid p \wedge {\sim}q) = 0$ hold by replacing *z* with *q* or *~q* in $P(p \rightarrow q \mid z) = P(q \mid p \wedge z)$.
  - For every sentence *r*, $P(r) = P(r \mid q) \cdot P(q) + P(r \mid {\sim}q) \cdot P({\sim}q)$ holds by expansion.
  - Taking *r* as $p \rightarrow q$, we have:
    - $P(r) = P(q \mid p)$, by $P(p \rightarrow q) = P(q \mid p)$.
    - $P(r \mid q) = P(q \mid p \wedge q) = 1$ and $P(r \mid {\sim}q) = P(q \mid p \wedge {\sim}q) = 0$, by $P(p \rightarrow q \mid q) = P(q \mid p \wedge q) = 1$ and $P(p \rightarrow q \mid {\sim}q) = P(q \mid p \wedge {\sim}q) = 0$.

  So:

    - $P(q \mid p) = 1 \cdot P(q) + 0 \cdot P({\sim}q) = P(q)$ holds by substitution on $P(r) = P(r \mid q) \cdot P(q) + P(r \mid {\sim}q) \cdot P({\sim}q)$.
  - *First conclusions:*
    - If $P(p \wedge q)$ and $P(p \wedge {\sim}q)$ are both positive then the propositions are probabilistically independent--that is absurd, though no contradictory.
    - Assigning standard true-values to any couple of propositions *p* and *q*, it derives that $P(q \mid p) = P(q)$, i.e. the conditional probability equals the probability of the consequent.

  Consequently:

- Any language expressing a conditional probability is a *trivial language*.

- Second Triviality Result:
  - Suppose that "→" is a probability conditional for a class of probability functions closed under conditionalizing, and take any probability function P in the class and any sentences p and q such that $(p \wedge q)$ and $P(p \wedge \sim q)$ are both positive. Proceeding as before, we have again:
    - $P(q \mid p) = P(q)$.
  - Take three pairwise incompatible sentences q, z and r such that $P(q)$, $P(z)$ and $P(r)$ are all positive. Replacing the disjunction $(q \vee z)$ with p, we have that $P(p \wedge q)$ and $P(p \wedge \sim q)$ are both positive, but $P(q \mid p)$ does not equal $P(q)$. This means there are no such three sentences.
  - *Second conclusions:*
    - P is a *trivial probability function* that never assigns positive probability to more than two incompatible alternative, so fixing at most four different values: $P(q)$=1 and $P(p)$=1--determining that $P(q \mid p)$ =1=$P(q)$--, $P(q)$=1 and $P(p)$=0--so that $P(q \mid p)$ is an undefined number--, $P(q)$=0 and $P(p)$=1--determining that $P(q \mid p)$ = 0 =$P(q)$--, $P(q)$=0 and $P(p)$=0--$P(q \mid p)$ is undefined again.

    Consequently:
    - For every class of probability functions closed under conditionalizing, "→" cannot be a probability conditional unless the class consists entirely of trivial probability functions.
    - Given that a probability function represents a possible system of belief, and some of such systems are not trivial, then indicative conditionals cannot be considered as probability conditionals for the whole class of probability functions.
    - It cannot be guaranteed that the probability of a conditional equals the corresponding conditional probability for all possible subjective probability functions, i.e. it is not a general rule that the absolute

probability of a conditional proposition equals the probability of its consequence on condition of its antecedent.

It is quite clear that the second Triviality Result logically entails the first one, reason for what Lewis' argument is called generally just "Triviality Result".

2. Dealing with trivialization: Stalnaker and Adams

The consequences generated by the Triviality Result could not have passed unnoticed in conditionals' debate, so that philosophers had to analyze their own thesis in front of this result. Hence, Stalnaker noticed that a previously coincidence, concerning simple conditionals, between his thesis and Adams' account cannot held in relation to compound sentences. However, he kept on considering conditionals as standard propositions, finally rejecting the Equation as a general principle. Particularly interesting in this regard is the letter written by Stalnaker to van Fraassen[80], in which he explicitly abandoned the idea to keep the Equation in a C2 system. Indeed, he presented an argument whose conclusion was the the same of Lewis, but by different assumptions. I shall reported it:

- Given any propositions A, B and C, and any probability function P, a sub-function P_A is a function defined for any P and a proposition A, such that: $P_A(B) = P(B|A)$, with $P(A) \neq 0$.
- Six thesis follow for reference:

  [1] If $P(A) \neq 0$, $P(A > B) = P(B|A)$.

  [2] Any sub-function is a probability function.

  [3] A "Metaphysical Realism", according to which the proposition expressed by a conditional sentence is independent with respect to the probability function defined on it.

  [4] If $P(A \wedge C) \neq 0$, $P(A > B \mid C) = P(B \mid A \wedge C)$.

[80] Robert Stalnaker, "Letter to van Fraassen", in *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, vol. 1, eds. W. Harper and C. Hooker (Dordrecht: Reidel, 1976), 302--306.

[5] The logic of the conditional is that one of C2.

[6] For any P, there are at most two disjoint propositions having a non-zero probability.

- Lewis derives [6] from [4], and [4] is a generalization of [1], by [3], in such a way:
  - $P_C(A \wedge B) = P_C(A) \cdot P_C(B|A) = P_C(A) \cdot P_C(A > B)$.
  - [3] permits that $P_C(A) \cdot P_C(A > B) = P(A|C) \cdot P_C(A > B|C)$--otherwise the conditional could not express the same proposition in both places.
  - $P_C(A \wedge B) = P(A \wedge B|C) = P(A|C) \cdot P(B|A \wedge C)$.
  - Assuming $P(A|C) \neq 0$, $P(A > B|C) = P(B|A \wedge C)$.
- Lewis' result--showing how to hold [1] would leads to [6]--depends on the assumption [3]. Therefore, technically, rejecting [3] the whole argument fails.
- However, there is another way to show the same conclusion, without involve [3] or [2]. That is showing how, assuming [1], [5] and the denial of [6], we have a contradiction:
  - By the denial of [6], we have at least three disjoint propositions to which some P assigns a non-zero probability. Take the propositions $A \wedge B$, $A \wedge \sim B$ and $\sim A$, abbreviating $A \vee (\sim A \wedge (A > \sim B))$ with C.
  - By [1] and [5] can be proved:

[7] If $P(\sim X) \neq 0$, then $P(X > Y|\sim X) = P(X > Y)$

[8] $\sim C$ entails $C > \sim(A \wedge \sim B)$

  - Since $\sim A$ entails $(\sim A \wedge (A > B)) \vee (\sim A \wedge (A > \sim B))$ and $P(\sim A) \neq 0$, then either $P(\sim A \wedge (A > B)) \neq 0$ or else $P(\sim A \wedge (A > \sim B)) \neq 0$.
  - It follows that:

[9] The propositions $A \wedge B$, $A \wedge \sim B$, $\sim A$, C and $\sim C$ have all non-zero probability.

  - Now:
    - By [8], $P(C > \sim(A \wedge \sim B)|\sim C) = 1$.
    - By [9], $P(\sim C) \neq 0$.
    - By [7], if $P(\sim C) \neq 0$ then $P(C > \sim(A \wedge \sim B)|\sim C) = P(C > \sim(A \wedge \sim B)) = 1$.

- By [9] again, $P(C)\neq 0$.
- By [1], If $P(C)\neq 0$, $P(C > \sim(A \wedge \sim B)) = P(\sim(A \wedge \sim B) \mid C)) = 1$.
- So, $P(\sim(A \wedge \sim B) \mid C)) = \frac{P(C \wedge \sim(A \wedge \sim B))}{P(C)} = 1$.
- It derives that $\frac{P((A \wedge B) \vee (\sim A \wedge (A > \sim B)))}{P((A \wedge B) \vee (A \vee \sim B) \vee (\sim A \wedge (A > \sim B)))} = 1$.
- Consequently, $P(A \wedge \sim B) = 0$, contradicting [9]!

So, Stalnaker concluded that, if we want to keep C2, avoiding troubles, we should reject the Equation. But it is not the only reason. Indeed, we could however decide to keep the Equation rather than C2. In that case, Stalnaker said we should know there are also several intuitive arguments against the thesis that the probability of a conditional equals the conditional probability.

Other philosophers, considering the Equation such as a great result in conditionals' treatment, preferred to preserve it, opting rather for trying to avoid Lewis' argument. First among all, there is Adams. He found in the Triviality Result an occasion for supporting his own thesis, recognizing it as a proof that indicative conditionals have not neither truth-values nor truth conditions. So, we must interpret their probability such as a conditional probability, not as probability of truth--otherwise we get a trivialization. Although, as previously anticipated, I am no totally convinced that Adams would have concluded in a so "drastically" way --rejecting any relation between truth and conditionals--unless any Triviality Result had been presented, it is unquestionable that he always used to spoke in terms of assertability rather than truth. Given that assertability generally goes with probability and the probability of a standard proposition is probability of truth, the reason for what this does not work with conditionals--how the Triviality Result showed--might be because they lack of truth-conditions. Therefore, Adams' suggestion to deny truth conditions and values looked practically *ad hoc*…but not for Lewis! Indeed, examining Adams' conclusion, Lewis claimed that it is actually invulnerable to the Triviality Result. This is because Adams, neither identifying conditional probability with probability of truth nor claiming that probabilities of conditional sentences obey to standard probability laws--but just to *assertability* laws--, made his hypothesis avoiding any proof built on the application of standard probability calculus to the probabilities of conditionals--like Triviality Result is. So, Lewis did not intend arguing against conditionals as lacking of truth-conditions, but he simply objected Adams' insistence to call such a probability just "probability". Instead, he should have denoted a different term, because that one universally evokes a probability obeying to the laws of standard probability calculus. Therefore, according to Lewis, a position *à la* Adams could be better expressed rejecting *either* truth-values *either* probability of the indicative conditionals.

Anyway, the real problem of Adams' conclusion concerns compound sentences. Indeed, even if he was right, and conditionals with truth-valued antecedent and consequent would be governed only by assertability rules--different from standard probability rules--, what about those conditionals compounded of conditional antecedent and consequent, lacking themselves of any value, condition and probability of truth? Adams should admit that the common idea according to which we can know compound sentences' truth conditions is by the truth conditions of their sub-sentences. But, how could it be possible when sub-sentences lack truth conditions? In that case we need something different from those assertability rules, because in front of this new evidence they are not able to show how compound sentences work. We need at least a new semantics containing special rules or anything else able to explain them.

So, although Lewis did not explicitly reject Adams' conclusion, it was pretty clear that he did not agree either. Rather, he thought that "fortunately a more conservative hypothesis is at hand"[81]: Grice's theory. Its conversational rules could be identified with those special rules useful to understand why assertability goes with conditional probability. So, basically Lewis was suggesting that we should start from something already known, rather than run into those complications Adams' hypothesis requires. For this reason he adopted the material conditional's truth conditions, explaining the discrepancy between its probability of truth and its

[81] David Lewis, "Probabilities of Conditionals and Conditional Probabilities", 137.

assertability by a Gricean implication. In a first moment Lewis talked about a *conversational* implication, but then he opted for Jackson's theory, in favor of a *conventional* one[82]:

"An indicative conditional is a truth-functional conditional that conventionally implicates robustness with respect to the antecedent. Therefore, an indicative conditional with antecedent *A* and consequent *C* is assertable iff (or to the extent that) the probabilities $P(A \supset C)$ and $P(A \supset C/A)$ both are high. If the second is high, the first will be too; and the second is high iff $P(C/A)$ is high; and that is the reason why the assertability of indicative conditionals goes by the corresponding conditional probability."[83]

3. Edgington's argument

Adams managed anyway--maybe in a too simplistic way, or maybe not--to avoid the Triviality Result and conserve the Equation, so catching the attention of many philosophers. Dorothy Edgington is certainly one that, among them, presented a great argument in his support.

She developed a less technical variant of Adams' hypothesis:

"We are frequently uncertain whether if A, B, and our efforts to reduce our uncertainty often terminate, at best, in the judgment that it is probable (or improbable) that if A, B. Of course, the truth-conditions theorist does not have to deny these undeniable facts. For him, to judge it more or less probable that if A, B is to judge it more or less probable that its truth conditions obtain. But this pinpoints his mistake. I show that uncertainty about a conditional is not uncertainty about the obtaining of any truth conditions. If a conditional had truth conditions, it would be. Therefore, a conditional does not have truth conditions." [84]

In support of the thesis that conditional sentences have not truth conditions of any kind, Edgington presented her famous arguments against the material conditional "⊃"--showing that it is generally weaker than the indicative conditional,

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[82] David Lewis, *Philosophical Paper: Volume II* (USA: Oxford University Press, 1987), 151--156.

[83] David Lewis, *Philosophical Paper: Volume II*, 153.

[84] Dorothy Edgington, "Do Conditionals Have Truth Conditions?", *Crítica* vol. 18 (1986): 6--7.

so that it would be wrong to identify "→" such as "⊃"--and against non-truth-functionality.[85]

First, Edgington showed that the well-known paradoxes of material conditional are not exactly overcome neither in a Gricean account. Indeed, according to her, Grice's idea to invoke the contrast between what is reasonable to believe and what is reasonable to assert is appropriate to explain just the disjunction, but not a material conditional. This is because such a contrast is not generally discernible when we have to do with conditional statement, showing a distinction between disjunctions and conditionals. So, whenever not prepared to reject the material conditional, we should accuse the speaker of inconsistent belief, although he really does not feel unreasonable at all. Of course, this is not the case of someone who is *totally certain* about a proposition--given that we would not assert any indicative conditional with a p antecedent when we are 100% certain about $\sim p$. But, on the other hand, if someone is 90% certain about $\sim p$, it is absolutely plausible to talk about what will be the case if p. In such a circumstance, according to the material conditional's account, the speaker must rationally be at least 90% certain of any conditional with a p antecedent. So, according to a ⊃-reading, it should be absurd to believe--even not with totally certain--that

[1] "Berlusconi will not win next elections" ($\sim p$)

is true and to reject that

[2] "If Berlusconi wins next elections, he will be a good president" ($p \rightarrow q$)

--given that "$p \supset q$" equals "$\sim p \vee q$".[86]

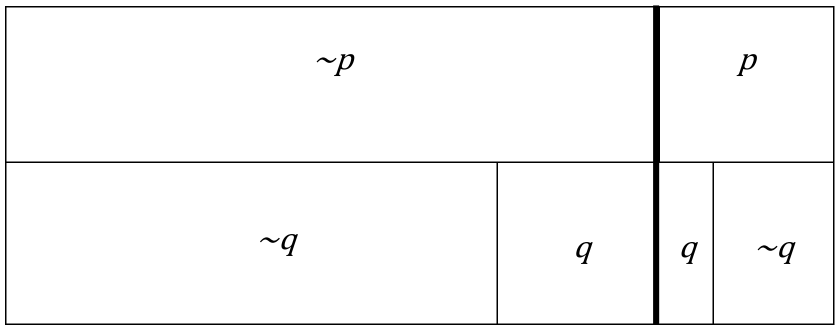
In other words, Edgington is saying it is too much--or at least weird--that the falsity of the antecedent makes true a conditional statement. That would be

[85] See Dorothy Edgington, "Do Conditionals Have Truth Conditions?", 3--39, and Dorothy Edgington, "On Conditionals", *Mind* vol. 104 (1995): 235--329.

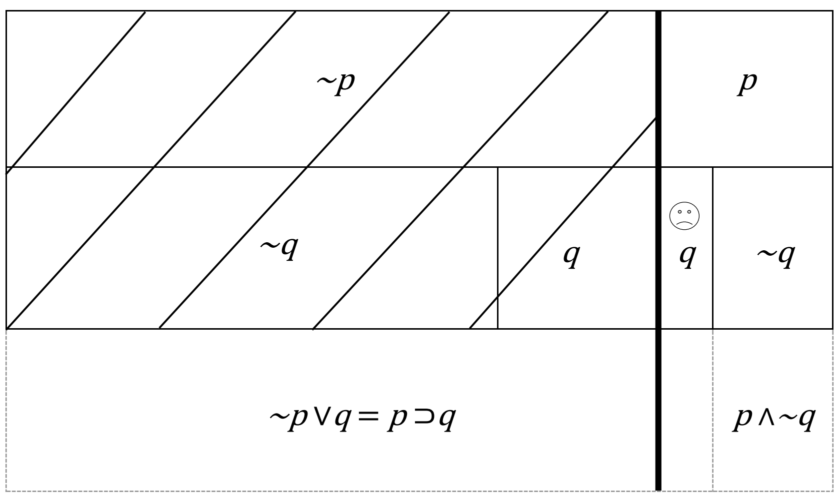
[86] This is a classic paradox of material conditional due to the fact that the antecedent's falsity always determines the truth of "⊃".

wonderful, but it does not work really! Hence, we need an interpretation of "→" stronger than a material conditional, able to work when "⊃" fails.

Edgington tried to explain which kind of reasoning we make when we accept [1]--because highly probable--and reject [2]. This is an example about how someone might consider different possibilities:



Now, analyzing [2], the speaker is assuming *p* and ignoring those possibilities concerning *~p:*



So, in Edgington's account, our belief on *q*--assuming *p*--is really low, leading the speaker to reject the conditional. In spite of that, treating [2] as $p \supset q$, his belief on the conditional would equal his belief on $\sim p \vee q$, which is highly probable.

In addition, the table below shows that, while the improbability of $p \wedge \sim q$ is a necessary and sufficient condition for the probability of the material conditional, this is not the case of Edgington's analysis--where it is a necessary but not a sufficient status.

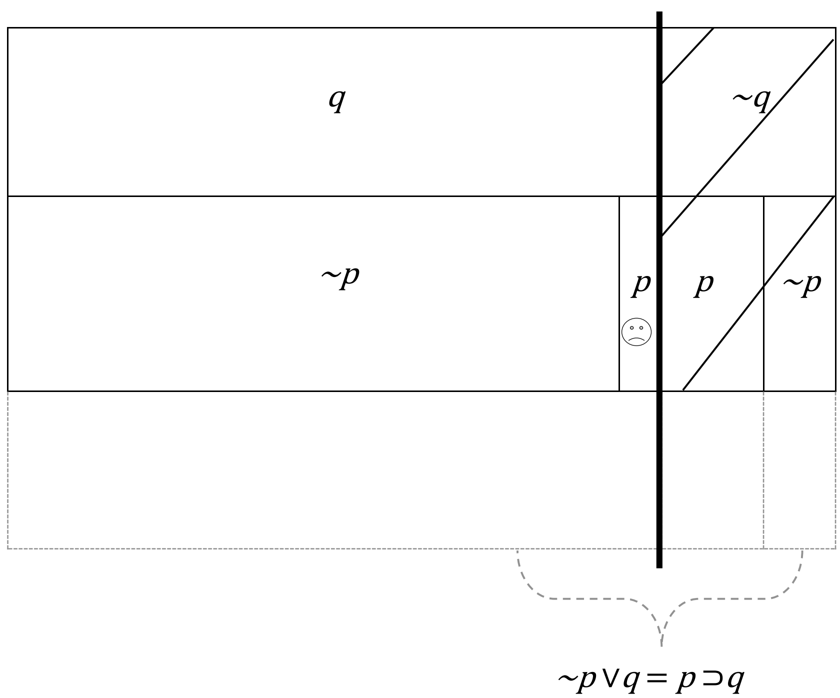
The same method can be used to consider another paradoxical case met by "⊃", according to which if we are 90% certain about *p* we have to highly believe in any conditional having *p* as its consequent. Hence, would be irrational to think--even not with totally certain--that

[3] "The actual government will win the next elections" (q)

is true but also that

[4] "If a financial scandal involving the actual President emerges, then the actual government will win the next elections" ($p \rightarrow q$)

is false.[87] On the other hand, it is perfectly coherent in Edgington's account:



[87] This is the other classic paradox of material conditional due to the fact that the consequent's truth always determines the falsity of "⊃".

If all of these arguments are not sufficient to discourage the reader to pursue a material conditional's account, Edgington also reminded us it does not work much better with compound sentences either.

In conclusion, she showed that to believe in an indicative conditional does not coincide with believing in the truth-functional conditional "⊃" , so that would be wrong to assign to "If *p*, *q*" the same truth conditions of $p \supset q$. This is because when we believe an indicative conditional we are considering how probable it is, on the supposition its antecedent is true. It follows that:

"X believes that (judges it likely that) if A, B, to the extent that he judges that A&B is nearly as likely as A or, roughly equivalently, to the extent he judges A&B to be more likely than A&~B."[88]

This means that we believe in an indicative conditional $p \rightarrow q$ when the ratio $\frac{P(p \wedge q)}{P(p)}$ is high, i.e. when its *conditional probability* is high.

But what about other kinds of truth conditions? Can a non-truth-functional account give a good reading of $p \rightarrow q$? Well, Edgington said that, however, there are more arguments in favor of her suppositional account rather than any non-truth-functional interpretation. Indeed, unlike non-truth-functionality, her interpretation can preserve the force of the standard truth-functionality guaranteeing that, given any two propositions *p* and *q*, the confidence in $p \vee q$ is sufficient for the certainty of $\sim p \rightarrow q$. This does not happen with non-truth-functionality. On the other hand, Edgington's account agrees with the non-truth-functional interpretation about the possibility of disbelieving either *p* either $p \rightarrow q$.

So, showing that in standard logic a suppositional view is not compatible with both truth-functional and non-truth-functional interpretations, Edgington concluded that there are no evidence to assign truth conditions of any kind to indicative conditionals. Consequently, the probability of a conditional *cannot* represent the probability that any proposition is true, but it is just the conditional probability $P(q \mid p)$.

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[88] Dorothy Edgington, "Do Conditionals Have Truth Conditions?", 17.

In such a way Edgington showed that Lewis' Triviality Result does not surprise at all, but it just represents a different argument demonstrating that $P(q|p)$ cannot be the truth-probability of any proposition. So, according to Edgington, the right position to adopt in front of the trivialization is conserving the Equation and denying any kind of truth-conditions for conditional sentences, like her arguments showed independently from Lewis' result.

IV

THE LOGIC OF TRIEVENTS

1. De Finetti's original trievents

Lewis' triviality Result had the consequence to split up the philosophical debate over conditional statements in two viewpoints: propositional or non-propositional.[89] This is because the common idea was that the only way to avoid a trivialization would be denying that a conditional is true or false. In other words, to elude Lewis' proof we should not treat conditional statements as standard propositions--having truth conditions. So, who is not prepare to assume a non-propositional position seems to have the only option to reject the Equation--$P(p \rightarrow q) = P(q \mid p)$, when $P(p) > 0$--assuming that conditionals always have truth conditions.

Given that Lewis' result depends on the assumption that conditional statements are *two-valued* propositions, my aim is to consider a third option questioning that a conditional can just have two values, but rather that it may be true, false or neither true nor false. In such a way it is not necessary, for avoiding the Triviality Result, to make a decision between to deny every kind of truth condition and to hold the Equation. Proving that such a different manner to elude any trivialization can be pursued, we do not have to renounce to assume a propositional viewpoint maintaining also the thesis that the probability of a conditional is its conditional probability.

The propose I will analyze is that one developed by Alberto Mura who, modifying the original de Finetti's three-valued semantics, provided a middle way between the above viewpoints. The intent is that of finding a new semantics able to incorporate Adam's logic such a fragment of a (three-valued) partial modal logic, helping in solving those problems related to compound and iterated conditionals.

[89] With the term "proposition" I mean a statement having in general a truth-value. In case of two-valued logic a proposition can just be true or false, while in a three-valued logic it can be true, false or even null.

I shall introduce, first, the original de Finetti's theory, fundamental for understanding Mura's contribute.

Bruno de Finetti is known to be the founder--together with Ramsey, but independently--of the subjective interpretation of probability. He developed an analysis in terms of a betting system: probability is a special case of prevision corresponding with the price of a bet. In case of a conditional bet, that is a gamble on a proposition *q* supposed that an event *p* happens, its price will equal the conditional probability of $q \mid p$, i.e. a conditional bet coincides with a suppositional conditional. [90]

According to de Finetti, a conditional bet on *q* supposed that *p* will be (i) win when either *p* either *q* are true, (ii) lost when *p* is true and *q* false, (iii) called off when *p* is false. Therefore, he suggested to assign to $q \mid p$ a truth-value just in case of win or loss, and to consider it *null*--neither true neither false--when the bet is cancelled. In such a way a conditional event appears as a three-valued proposition, called "trievent".

So, in 1935, de Finetti proposed a kind of logic of conditional events, known as "Logic of Trievents", consisting in a three-valued logic expressing in a significant form the question concerning conditional probabilities.[91] The basically idea is that the act to assume a standard two-valued logic is just a conventional issue: propositions are not true or false because of *a priori* principle, but because we conventionally decided to call "propositions" those logical entities needing of a "yes"

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[90] De Finetti made use of the notion of "conditional expectation"--$P(X \mid H) = P(X \wedge H)/P(H)$--introduced by himself in 1931, that allows to interpret the conditional probability such as the expected conditional value of the prize of a conditional bet. This is important because Stalnaker & Jeffrey and McGee made the mistake to consider the value of a conditional bet such as the absolute expectation value of its prize, interpreting a called off bet such as zero profit. But in the Bayesian theory a called off bet is something which remains unchanged to positive linear transformations--there is not any zero equipped of an intrinsic value.

[91] Bruno de Finetti (1935), "The Logic of Probability", *Philosophical Studies* 77, (Netherlands: Kluwer Academic Publishers, 1995), 181--190. Translated by R. B. Angell from Bruno de Finetti, "La logique de la probabilité", in *Actes du Congrès International de Philosophie Scientifique* (Sorbonne, Paris: Hermann Éditeurs, 1936), IV 1--9.

or "no" as answer. But, if we agreed on assume three values, we could have an analogue of standard logic, but with more values, differing just in a purely formal way.

In the Logic of Trievents the third value is not, strictly speaking, a value like "true" or "false". It should be considered as a third possible *attitude* that someone can adopt toward a proposition when he is in doubt between answering "yes" or "no". In other word, this third value is void--or *null*--and can be understood as a *gap*. However, a null event is something different from an indeterminate event *á la* Łukasiewicz--whose truth conditions are unknown. Rather, de Finetti meant an event whose conditions under which it would be true or false are not satisfied. We can find several de Finetti's papers talking about this third value and he has never changed his interpretation about that. It is particularly interesting the passage in which he identified a null event with an "aborted event":

> "If a distinction results in being incomplete, no harm done: it would mean that besides "true" and "false" events I would also have "null" events, or, so to speak, aborted events. As a matter of fact, it is sometimes useful to consider explicitly and intentionally from the very start such a "trievent" (especially, as will be seen later, with respect to probability theory). If, for instance, I say: "supposing that I miss the train, I shall live by car", I am formulating a "trievent", wich will be either true or false if, after missing the train, I leave by car or not, and it will be null if I do not miss the train."[92]

Standard logic's truth-tables can be expanded to include the null value in such a way:

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[92] Translation by Alberto Mura of Bruno de Finetti (1934), *L'invenzione della verità* (Milano: Cortina, 2006), 103, in: Alberto Mura, "Probability and the Logic of de Finetti's Trievents", in *Bruno de Finetti Radical Probabilist*, ed. M. C. Galavotti (London: College Publications, 2009), 204.

P	*q*	$\sim p$	$p \vee q$	$p \wedge q$	$p \supset q$[93]	$q \mid p$
T	**T**	F	T	T	T	T
T	**F**	F	T	F	F	F
T	**N**	F	T	N	N	N
F	**T**	T	T	F	T	N
F	**F**	T	F	F	T	N
F	**N**	T	N	F	T	N
N	**T**	N	T	N	T	N
N	**F**	N	N	F	N	N
N	**N**	N	N	N	N	N

While conjunction and disjunction basically coincide with those ones proposed by Łukasiewicz' three-valued logic, conditioning is the new truth-function introduced by de Finetti. So, the real innovation consists just in the truth-functional connective "|".

According to de Finetti, such a kind of logic should help us to manage those troubles due to a two-valued analysis, with the advantage that every proposition can be translated in terms of standard logic--given that every trievent is a simply formal representation of pairs of ordinary events.[94] Indeed, a return from the Logic of Trievents to the standard two-valued logic is possible by the introduction of two

[93] This material conditional is today known as "Kleene's strong material implication", because independently proposed later by Kleene in 1938. See Stephen C. Kleene, "On Notation for Ordinal Numbers", *The Journal of Symbolic Logic*, Vol. 3, No. 4 (1938): 150--155.

[94] However, it should be pointed out that the algebra of such a pairs of ordinary events--isomorphic to the trievents' algebra--is not Boolean. It is rather a distributive lattice that does not admit a unique complement--it means it does not hold CEM.

operations: *thesis* (T) and *hypothesis* (H).[95] $T(X)$ means "X is true" and $H(X)$ means "X is not null":[96]

X	$T(X)$	$H(X)$
T	T	T
F	F	T
N	F	F

The above truth-table shows it holds that $X = T(X) \mid H(X)$, i.e. every trievent ϕ is true, given that it is not null. This result is known as "de Finetti's Decomposition Theorem".[97]

Given that every trievent can be represented by any conditional event $q \mid p$--where p and q are ordinary events--, for the Decomposition Theorem it holds that $q \mid p = T(q \mid p) \mid H(q \mid p)$. Looking at the truth-table of "|", excluding those cases where p and q are aborted events, the Decomposition Theorem leads to two important consequences:

- $q \mid p$ is true if and only if both p and q are true--$T(q \mid p) = p \wedge q$.
- $q \mid p$ is not null if and only if p is a tautology--$H(q \mid p) = p$.

p	q	$q \mid p$
T	**T**	**T**
T	**F**	**F**
F	T	N
F	F	N

[95] The rule of Thesis and Hypothesis is *just* that of allowing a conversion into standard logic. So, technically, they are not operators belonging to the logic of Trievents. About this, see Alberto Mura, "Probability and the Logic of de Finetti's Trievents", in *Bruno de Finetti, Radical Probabilist*, ed. M. C. Galavotti (London: College Publications, 2009), 207--209.

[96] In terms of betting system, "the 'thesis' of the tri-event, is the case in which one has established that the bet is won; the 'hypothesis' the case in which one has established that the best is in effect". Bruno de Finetti (1935), "The Logic of Probability", 186.

[97] So called by Alberto Mura. See Alberto Mura, "Probability and the Logic of de Finetti's Trievents", 208.

Thus, it results that $q|p = T(q|p) | H(q|p) = (p \wedge q) | p$.

If p is not a tautology it means it could be false, so that $q|p$ is not an ordinary event. Consequently, an ordinary event is nothing less than a particular case of a trievent when p is a tautology. Therefore, "to introduce the notion of conditional probability is to extend the definition of $P(X)$ from the field of ordinary events, X, to the field of tri-events".[98] In Mura 2009 we can find two methods to obtain such extension[99]:

- First, a probability function on a Boolean algebra $\mathscr{B}$ of ordinary events has to be defined, and then it can be extend to a quotient lattice $\mathscr{L}$ by de Finetti's Decomposition Theorem, because for every element X of $\mathscr{L}$ there are two element, p and q, in $\mathscr{B}$ such that $X = q|p$. Assumed the last result, an extension is given simply in such a way: $P(q|p) = P(p \wedge q) | P(p)$--provided $P(p)>0$.
- Alternatively, a probability function can be defined directly on $\mathscr{L}$--so that it will remain however defined on $\mathscr{B}$, because it is contained in $\mathscr{L}$--in such a way:
  - Representing the original de Finetti's operations $T(X)$ and $H(X)$ respectively by the symbols ↑ and ↕, it holds that $\uparrow X =$ "X is true" and $\updownarrow X =$ "X is not null".
  - Be $\boldsymbol{P}$ a partially probability function such that $\boldsymbol{P}(X)$ is not defined if and only if $\boldsymbol{P}(\updownarrow X)=0$.
  - the following axioms holds:
    [A1]. If $\boldsymbol{P}(\updownarrow p)>0$ then $\boldsymbol{P}(p)\geq 0$.
    [A2]. $\boldsymbol{P}(\top)=1$.

[98] Bruno de Finetti (1935), "The Logic of Probability", 184--185.

[99] Alberto Mura, "Probability and the Logic of de Finetti's Trievents", 214--216.

[A3]. If $\boldsymbol{P}(\updownarrow p)>0$ then $\boldsymbol{P}(p)=\frac{\boldsymbol{P}(\uparrow p)}{\boldsymbol{P}(\updownarrow p)}$[100].

[A4]. $\boldsymbol{P}(\uparrow(p\vee q))=\boldsymbol{P}(\uparrow p)+\boldsymbol{P}(\uparrow q)-\boldsymbol{P}(\uparrow(p\wedge q))$.

[A5]. If $\boldsymbol{P}(\updownarrow p)>0$ then $\boldsymbol{P}(\sim p)=1-\boldsymbol{P}(p)$.

- Proving the following theorem:
  - if $\boldsymbol{P}$ is a probability function defined over $\mathscr{B}$ and if, for every $X\in\mathscr{L}$, it holds that $\boldsymbol{P}(\updownarrow X)>0$, then there exist two elements p and $q\in\mathscr{B}$ such that $X=q\,|\,p$ and $\boldsymbol{P}(X)=\frac{\boldsymbol{P}(p\wedge q)}{\boldsymbol{P}(p)}$

  we are also demonstrating that the above axioms provide the same class of functions obtained by the first extension method.

- Proof of the above theorem:
  - Let p and q be any element of $\mathscr{B}$ and let $\boldsymbol{P}_{\mathscr{B}}$ a probability function defined on $\mathcal{B}$:
    - [A1] and [A2] are trivially satisfied.
    - Since $p\in\mathscr{B}$, it holds that $p=\uparrow p$ and $\updownarrow p=\top$.
    - Since $\boldsymbol{P}(\top)=1$, [A3] is trivially satisfied too.
    - Since $\uparrow(p\vee q)=(p\vee q)$, $\uparrow p=p$, $\uparrow q=q$ and $\uparrow(p\wedge q)=(p\wedge q)$, [A4] is obviously satisfied.
    - Since $\updownarrow p=\top$, $\boldsymbol{P}(\updownarrow p)=1$ so that [A5] equals, with respect to the element of $\mathscr{B}$, the axiom of complement. Hence, [A5] is satisfied as well.
    - Given that $\boldsymbol{P}_{\mathscr{B}}$ satisfies every axioms [A1]—[A5], then it is a probability function defined on $\mathscr{B}$.
  - Let $X\in\mathscr{L}$ be such that $X=q\,|\,p$--where p, $q\in\mathscr{B}$:
    - By [A3] it holds that $\boldsymbol{P}(X)=\boldsymbol{P}(q\,|\,p)=\frac{\boldsymbol{P}(\uparrow(q\,|\,p))}{\boldsymbol{P}(\updownarrow(q\,|\,p))}$--provided $\boldsymbol{P}(\updownarrow(q\,|\,p))>0$.

[100] A3 shows that "the probability of trievents depends functionally on the probability of ordinary events. Without reference to ordinary events, no set of probability axioms are therefore possible for trievents". Alberto Mura, "Probability and the Logic of de Finetti's Trievents", 216.

- Given that $\uparrow(q \mid p) = \uparrow(p \wedge q)$, and since $p, q \in \mathscr{B}$, it holds that $\uparrow(p \wedge q) = (p \wedge q)$.
- Given that $\updownarrow(q \mid p) = (\uparrow p \wedge \updownarrow q)$, and since $p, q \in \mathscr{B}$, it holds that $\updownarrow q = \top$ and $\uparrow p = p$.
- Therefore, $\updownarrow(q \mid p) = p \wedge \top = p$.
- By substitution, it holds that $\boldsymbol{P}(X) = \frac{\boldsymbol{P}(\uparrow(q \mid p))}{\boldsymbol{P}(\updownarrow(q \mid p))} = \frac{\boldsymbol{P}(p \wedge q)}{\boldsymbol{P}(p)}$. So, axioms [A1]—[A5] provides the same class of function obtained by the extension method from a Boolean algebra $\mathscr{B}$ to a quotient lattice $\mathscr{L}$.

In conclusion, de Finetti's analysis shows that every probability function defined on a Boolean algebra of ordinary events can be univocally extended to the whole trievents lattice, so that, given two standard proposition p and q, the probability of the trievent $q \mid p$ equals the ratio between the probability of $p \wedge q$ and the probability of p. Consequently, "|" appears such as a connective satisfying the Equation but with the advantage of avoiding Lewis' Triviality Result--because $q \mid p$ is not an ordinary event, but a three-valued proposition.

2. Avoiding Trivialization

Basically, Lewis' Triviality Result derives from these assumptions:

(i) $P(p \rightarrow q \mid z) = P((q \mid p) \mid z) = P(q \mid (p \wedge z))$--with $P(p \wedge z) > 0$.

(ii) $P(r) = P(r \mid q) \cdot P(q) + P(r \mid {\sim}q) \cdot P({\sim}q)$.

Both entail the trivialization:

(iii) $P(q \mid p) = P(q)$.[101]

Therefore, conditioning would be satisfied just in a few special (banal) cases so that, technically, "|" cannot be a standard truth-functional iterable connective. But, what about considering r as a trievent?

Although (i) is generally satisfied by de Finetti's conditioning, (ii) is not. Indeed:

[101] Replacing z with q or ${\sim}q$ it holds that $P(q \mid (p \wedge q)) = 1$ and $P(q \mid (p \wedge {\sim}q)) = 0$. Replacing r as $q \mid p$ we have that $P(q \mid p) = P((q \mid p) \mid q) \cdot P(q) + P((q \mid p) \mid {\sim}q) \cdot P({\sim}q)$. By substitution, it holds: $P(q \mid p) = P(q \mid (p \wedge q)) \cdot P(q) + P(q \mid (p \wedge {\sim}q)) \cdot P({\sim}q) = P(q)$.

- By [A3] we obtain the following generalization of (ii):

(iv) $P(r) = \frac{P(\uparrow r)}{P(\updownarrow r)} = \frac{P(\uparrow r \mid \uparrow q) \cdot P(\uparrow q) + P(\uparrow r \mid \sim\updownarrow q) \cdot P(\sim\updownarrow q) + P(\uparrow r \mid \uparrow\sim q) \cdot P(\uparrow\sim q)}{P(\updownarrow r)}$ [102].

- (iv) can be reduced to (ii) when, for q and r both belonging to $\mathscr{B}$, $P(\updownarrow r) = P(\top) = 1$, $P(\sim\updownarrow q) = P(\bot) = 0$ and $P(\uparrow r) = P(r)$.
- There is no reason for requiring that (ii) would be in general satisfied by trievents, given that it is just a special case--with n=2--of the so called *Conglomerative Property*, according to which, for every finite partition of ordinary events $q_1,\dots, q_n$ such that $q_1 \wedge q_j = \bot$ $(1 \leq i < j \leq n)$ and $q_1 \vee \dots \vee q_n = \top$, it holds:

(v) $P(r) = P(r \mid q_1) \cdot P(q_1) + \dots + P(r \mid \sim q_n) \cdot P(\sim q_n)$[103].

- (iv) is a special case of (v) too--with n=3--and it is exactly the generalization of (ii) for trievents. Indeed, we can naturally represent in $\mathscr{B}$ any ordinary event p by a partition of two elements--$\{p, \sim p\} = \{\uparrow p, \uparrow\sim p\}$--and any trievent q by a partition of three elements--$\{\uparrow q, \sim\updownarrow q, \uparrow\sim q\}$. This is also confirmed by the theorem according to which:
    - Let p, q, r be three elements in $\mathscr{B}$ forming a partition and let that $K = \{y \in \mathscr{L} \mid \uparrow y = p, \uparrow\sim y = q, \sim\updownarrow y = r\}$. Satisfying such a conditions, K contains just one element.
  - *Proof:*
    - Let **V** be the set of all valuations V: $\mathscr{L} \rightarrow$ {t, f, n} and let $y = p \mid (p \vee \sim r)$.
    - For every valuation v in **V** it holds: $v(p)$=t iff $v(y)$=t--so that $\uparrow y = \uparrow p = p$--, $v(q)$=t iff $v(y)$=f--so that $\uparrow\sim y = q$--, and $v(r)$=t iff $v(y)$=n--so that $\sim\updownarrow y = r$.
    - $y \in K$ and $K \neq \emptyset$.

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[102] To be clear: $\updownarrow X$= "X is not null"; $\sim\updownarrow X$= "X is null"; $\uparrow X$= "X is true"; $\uparrow\sim X$= "X is false"; $\sim\uparrow X$= "q is not true" = "X is null or false".

[103] (v) derives from:

- $P(r \mid q) = \frac{P(r \wedge q)}{P(q)}$; $P(r \wedge q) = P(r \mid q) \cdot P(q)$.
- If $q_1, \dots, q_n$ is exhaustive and mutually exclusive, then $P(r) = P(r \wedge q_1) + \dots + P(r \wedge q_n)$.

- Suppose that both w_1 and w_2 belong to $\mathscr{L}$, such that: $\uparrow w_1 = \uparrow w_2 = p$; $\uparrow\sim w_1 = \uparrow\sim w_2 = q$; $\downarrow w_1 = \downarrow w_2 = r$.
- Suppose that, for some v in **V**, it holds $v(w_1) \neq v(w_2)$.
- It holds that: if $v(w_1)$=t then $v(\uparrow w_1)$=t and $v(\uparrow w_2)$=f--against the hypothesis--; if $v(w_1)$=f then $v(\uparrow\sim w_1)$=t and $v(\uparrow\sim w_2)$=f--against the hypothesis--; if $v(w_1)$=n then $v(\sim\updownarrow w_1)$=t and $v(\sim\updownarrow w_2)$=f--against the hypothesis again. Therefore, $w_1 = w_2$.

So, we just need to notice that $\uparrow y$, $\uparrow\sim y$ and $\sim\updownarrow y$ all belong to $\mathscr{B}$ and form a partition, to prove the theorem according to which:

- For every trievent $y \in \mathscr{L}$ there exists a partition of events p, q, r belonging to $\mathscr{B}$ such that $p = \uparrow y$, $q = \uparrow\sim y$, $r = \sim\updownarrow y$.

In conclusion, it seems de Finetti knew clearly that, if p and q are two events satisfying the excluded middle law, then $q\,|\,p$ cannot be interpreted as such event. But, considering it as a trievent, then "|" appears as a suppositional connective able to represent in a good way the conditional probability.

3. De Finetti's difficulties

Although de Finetti's account can represent a way to avoid trivialization conserving the Equation it is not free from problems, making it unable to provide a right semantic for conditional statements.

Firstly, the correspondence between logic and probability, in spite of increasing in relation to some aspects, loses some properties on the other side. Among them, the fact that in de Finetti's account $\phi\,|\,\phi$ is not a tautology, but a *quasi-tautology*, because although it is not false it can be either true or null. So, given any p and q and any probability function $\boldsymbol{P}$, if $\boldsymbol{P}(p) = \boldsymbol{P}(q)$ but $p\,|\,p$ is not truth-functionally equivalent to $q\,|\,q$, then $p \neq q$. In other words, it does not work the propriety according to which, when two trievents have same probability, the respective propositions are logically equivalent. This means there are a variety of trievents, to which every probability function assigns probability 1, but not logically equivalent.

Similarly, any de Finetti's contradiction is a *quasi-contradiction*[104], given that it cannot be true, but can be either false or null. So, it is easy to catch that there are some elements able to be quasi-tautologies and quasi-contradictions at the same time.

Now, what is wrong with quasi-tautologies and quasi-contradictions? Classical probability, defined on a Boolean algebra, is shown as a generalization of propositional logic when, for every probability function P, the following properties are satisfied:

- $p \leq q$ if and only if $P(p) \leq P(q)$.
- If $P(p) = P(q)$ then $p = q$.

While the first property is easily satisfied by the probability of trievents[105], the second one is not, because the only fact that $\boldsymbol{P}(p) = \boldsymbol{P}(q)$ is not sufficient to guarantee that $p = q$. And it is not sufficient just because every tautology and every contradiction in a trievents account are quasi-tautologies and quasi-contradictions. Hence, probability can appears as a generalization of propositional logic in trievents only at the prize of losing the property according to which if $\boldsymbol{P}(p) = \boldsymbol{P}(q)$ for every probability function then $p = q$. However, we can consider as tautologies those trievents which are true in every case in which they are not null. That is, ϕ is a tautology if it is true supposed it is not null--$\uparrow\phi \mid \updownarrow\phi$.[106]

Concerning quasi-tautologies and quasi-contradictions in a betting system, a conditional bet on them is a quite degenerate bet, because it would be a gamble without a real risk. Indeed, a bet on a quasi-tautology cannot be lost and a bet on a quasi-contradiction cannot be won. But, differently from a bet on a standard tautology or contradiction, it ca be called off. If this peculiarity seems justify *prima*

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[104] Both terms of "quasi-tautology" and "quasi-contradiction" due to Bergmann. See Merrie Bergman, *An Introduction to Many-Valued and Fuzzy Logic*, (Cambridge, UK: Cambridge University Press, 2008), 85--86.

[105] $p \leq q$ iff $\boldsymbol{P}(p) \leq \boldsymbol{P}(q)$ for every $\boldsymbol{P}$ such that $\boldsymbol{P}(\updownarrow p)>0$ and $\boldsymbol{P}(\updownarrow q)>0$. Alberto Mura, "Probability and the Logic of de Finetti's Trievents", 219.

[106] The identity $\varphi = (\uparrow\varphi \mid \updownarrow\varphi)$ always holds for every φ by the de Finetti's Decomposition Theorem.

facie the difference in a partial order, actually such a difference is not maintained when we coherently set on a bet. Indeed, if I set the same amount of money on two different trievents p and q, technically, they should be considered as the same trievent. In addition, even the difference between quasi-tautologies (or quasi-contradictions) and standard tautologies (or standard contradictions) is lost in a coherent bet: the profit I can have by a bet on ⊤ is the same of that one I can have by betting on a trievent $\phi | \phi$.

In conditional logic the limit of de Finetti's account concerns giving a definition of logical consequence in accordance with Adams' logic. Since Adams was able to extend such a notion from propositional logic to the logic of simple conditionals in perfect conformity with intuition[107]--except some very artificial cases--, it seems absolutely reasonable to request that a good semantic for conditional sentences be able to provide a notion of logical consequence in accordance with Adams' definition.[108]

Actually, de Finetti did not treat this point, but we can easily guess such notion in relation with his ideas. So, a logical consequence in a trievents account should preserve the property that holds in standard logic, according to which:

- $p \vDash q$ and $q \vDash p$ if and only if $p \cong q$, i.e. when two propositions p and q entail each other they have the same truth conditions and content.

To maintain this property in trievents, either truth either non-falsehood has to be conserved. Therefore, q is a logical consequence of p if and only if, for every evaluation v, the following conditions are preserved:

- *Preservation of truth:* if $v(p)$=t then $v(q)$=t.
- *Preservation of non-falsehood:* if $v(p) \in$ {t, n} then $v(q) \in$ {t, n}.

Now, a notion of logical consequence in accordance with Adams' logic should meet the following property:

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[107] See [THEOREM 3] and [THEOREM 4], 24--25.

[108] In addition, we should not forget that, concerning simple conditionals, Adams' conditional logic basically coincides with Stalnaker and Lewis's proposals.

- A sentence ϕ is a logical consequence of a set of sentences $\{\phi_1, \ldots, \phi_n\}$ if and only if the uncertainty of ϕ is not higher than the sum of the uncertainties of $\phi_1, \ldots, \phi_n$, i.e. if and only if $u(\phi) \leq u(\phi_1) + \ldots + u(\phi_n)$.

But, unfortunately, McGee 1981[109] presented a result, then generalized by Adams 1995[110], showing that there is no truth-functional many-valued logic--included de Finetti's Trievents--able to preserve Adams' definition of logical consequence.[111] Indeed, McGee's result showed that "*p-validity cannot be characterized by a finite matrix*, i.e. one cannot describe p-validity as the preservation of a finite set of designated values."[112] This is the proof he advanced:

- Given a finite matrix $\mathscr{M} = \langle M, D, +, \cdot, -, \uparrow \rangle$ meeting probabilistic logic--by reduction ad absurdum--, (i) $D \subseteq M$ is the set of designated values, (ii) $+, \cdot, -$ and $\uparrow$ are operations on M, (iii) M has n members and (iv) τ is a truth assignment satisfying the following conditions:
  - $\tau(\phi \vee \psi) = \tau(\phi) + \tau(\psi)$.
  - $\tau(\phi \wedge \psi) = \tau(\phi) \cdot \tau(\psi)$.
  - $\tau(\sim\phi) = - \tau(\phi)$.
  - $\tau(\phi \rightarrow \psi) = \tau(\phi) \uparrow \tau(\psi)$.
- Considering $\alpha_0, \ldots, \alpha_n$ such as distinct atomic sentences, the inference below is not probabilistically valid:

$\alpha_0 \rightarrow \sim\alpha_1 \wedge \sim\alpha_2 \wedge \ldots \wedge \sim\alpha_n$

$\alpha_1 \rightarrow \sim\alpha_2 \wedge \sim\alpha_3 \wedge \ldots \wedge \sim\alpha_n$

$\alpha_{n-1} \rightarrow \sim\alpha_n$

$\therefore\ \alpha_n \rightarrow \sim\alpha_0 \vee \sim\alpha_1 \vee \ldots \vee \sim\alpha_n$

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[109] Van McGee, "Finite Matrices and the Logic of Conditionals", in *Journal of Philosophical logic*, vol. 10 (1981): 349--351.

[110] Ernest W. Adams, "Remarks on a Theorem of McGee", in *Journal of Philosophical Logic*, vol. 24 (1995): 343--348.

[111] It would mean that no three-valued logic can entail Adams' p-validity.

[112] Moritz Schulz, "A note on two theorems by Adams and McGee", in *The Review of Symbolic Logic*, vol. 2 (2009): 510.

This means there is a truth assignment τ giving a designated value to every premises and an undesignated value to the conclusion.

- Given that M has only n members, there are i and j such that $0 \leq i \leq j \leq n$ and $\tau(\alpha_i) = \tau(\alpha_j)$. Now it holds that:

  $\tau(\alpha_i \rightarrow \sim\alpha_{i+1} \wedge \ldots \wedge \sim\alpha_{j-1} \wedge \sim\alpha_i \wedge \sim\alpha_{j+1} \wedge \ldots \wedge \sim\alpha_n) =$

  $= \tau(\alpha_i) \uparrow (-\tau(\alpha_{i+1}) \cdot \ldots \cdot -\tau(\alpha_{j-1}) \cdot -\tau(\alpha_i) \cdot -\tau(\alpha_{j+1}) \cdot \ldots \cdot -\tau(\alpha_n)) =$

  $= \tau(\alpha_i) \uparrow (-\tau(\alpha_{i+1}) \cdot \ldots \cdot -\tau(\alpha_{j-1}) \cdot -\tau(\alpha_j) \cdot -\tau(\alpha_{j+1}) \cdot \ldots \cdot -\tau(\alpha_n)) =$

  $= \tau(\alpha_i \rightarrow \sim\alpha_{i+1} \wedge \ldots \wedge \sim\alpha_{j-1} \wedge \sim\alpha_j \wedge \sim\alpha_{j+1} \wedge \ldots \wedge \sim\alpha_n) \in D.$

- So, the premise of the argument

  $\alpha_i \rightarrow \sim\alpha_{i+1} \wedge \ldots \wedge \sim\alpha_{j-1} \wedge \sim\alpha_i \wedge \sim\alpha_{j+1} \wedge \ldots \wedge \sim\alpha_n$

  $\therefore \alpha_n \rightarrow \sim\alpha_0 \vee \ldots \vee \sim\alpha_i \vee \ldots \vee \sim\alpha_n$

  has a designated value although its conclusion has an undesignated value. But such argument is probabilistically valid. Therefore, there is a contradiction!

Adams identified the point of such a result in the reason that any many-valued logic meets a principle known as *condensation property:*

"if replacing more than n distinct sentential variables in an inference by at most n distinct variables ('condensing' them) always results in an inference that is valid in the sense of many-valued logic, then the original inference must be valid in this sense. Since this is not the case in any of the conditional logics, they cannot be equivalent to any many-valued logics with finitely many values, no matter how they define the conditional."[113]

Finally, I would like to report and analyze Bradley's criticism about some three-valued approach--included de Finetti's one--in conditional treatment.[114] I will show that, actually, de Finetti's Trivents are not totally vulnerable to Bradley's argument. Indeed, two of the three counterexamples he presented--trying to show that a trivalent proposal would be definitely "hopeless"--do not hold with de Finetti's account.

[113] Ernest W. Adams, "Remarks on a Theorem of McGee", 343.

[114] Richard Bradley, "Indicative Conditionals", *Erkenntnis*, 56 (2002): 345--378.

Basically, Bradley identified the limit of a three-valued approach in:

(i) The interpretation of conditionals' conjunctions entails that $(p \rightarrow q) \wedge (\sim p \rightarrow q)$ is never true.

(ii) The standard treatment of negation makes the negated conjunct $\sim((p \rightarrow q) \wedge (\sim p \rightarrow z))$ equivalent to the conjunct $(p \rightarrow \sim q) \wedge (\sim p \rightarrow \sim z)$.

(iii) It holds the equivalence $(p \rightarrow q) \wedge (\sim p \rightarrow z) = (p \wedge q) \vee (\sim p \wedge q)$.

Now, even if $(p \rightarrow q) \wedge (\sim p \rightarrow q)$ is never false, the fact that it is never true certainly makes sentences like "If it rains, the match will be played and, if it does not rain, the match will be played (as well)" always null. However, (i) come from the fact that Bradley assumes the introduction rule for the conjunction, according to which: $p, q \vdash p \wedge q$. But, such a rule does not hold in general for Trievents, so that asserting $\{p, q\}$ does not mean to assert $(p \wedge q)$. As well, denying a set does not mean to deny its elements, but simply no asserting them all together. So that, also (ii) does not hold unless assuming the introduction rule for the conjunction.

Although (i) and (ii) do not work with Trievents, (iii) actually does. So, the sentence $(p \rightarrow q) \wedge (\sim p \rightarrow z)$ is equivalent to $(p \wedge q) \vee (\sim p \wedge q)$, and this does not generally hold in natural language. Indeed, according to Bradley, two sentences with same truth-value are not necessarily the same thing, given that they can have different meaning. This is because Bradley denies that the content of a conditional is characterized just by its truth-value. For such a reason propositions with same truth-value do not have always same probability--leading Adams to conclude that the probability of a conditional is not his probability of truth.

However, a proposal by Alberto Mura, aiming to solve those problems of de Finetti's original trievents, seems to overcome also (iii)--that does not hold generally in Mura's account. In such a way, given that trievents can avoid a trivialization, we should be able to either preserve the Equation either fix conditionals' truth conditions.

V

MODIFIDING TRIEVENTS

1. Mura's Semantics of Hypervaluations

In 2009, Alberto Mura elaborated a modified account of de Finetti's Trievents, called Semantics of Hypervaluations, with the intent of providing a new semantics for Adams' conditional logic.[115]

A hypervaluation is a two-stage valuation, introducing a modal component in the second stage. Although a great similarity between it and Van Fraassen's supervaluation[116], there are some important differences:

- The original motivation to develop a supervaluated account was to save classical tautologies, which kept on being supervaluated as true--and every contradiction as false. Instead, the hypervaluations need to distinguish between classical tautologies and quasi-tautologies, because of the null value. Such a difference cannot pertain to the supervaluations because in Kleene's logic we never can obtain *n*. The hypervaluations allow to evaluate every non-null quasi-tautology as a tautology, called *pre-tautology*.
- Because in Kleene's logic there is not the connective "|", a sentence $\phi|\phi$ has to be evaluated in a standard way by supervaluations. Contrary, de Finetti's account provides "|" so that, according to hypervaluations, $\phi|\phi$ is a quasi-tautology.
- While hypervaluations are compositional in character, supervaluations are not. Indeed, a classical tautology evaluated as a tautology by supervaluations does not work as a tautology in compound sentences.

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[115] Alberto Mura, "Probability and the Logic of de Finetti's Trievents", in *Bruno de Finetti, Radical Probabilist*, ed. M. C. Galavotti (London: College Publications, 2009), 201--242.

[116] See Bas Van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic", *Journal of Philosophy* 68 (1966): 481--495, and Bas Van Fraassen, "Hidden Variables in Conditional Logic", *Theoria*, vol. 40 (1974): 176--190.

For example, even if $\phi \vee \sim\phi$ is always true by every supervaluation, nonetheless $(\phi \vee \sim\phi) \wedge \psi$ is not classically evaluated as ψ. This kind of problem does not concern the hypervaluations, because they use to work directly on pre-tautologies, considering them as tautologies keeping on holding even in compound sentences. This compositional character of hypervaluations makes them recursively defined.

All of these points make the supervaluations not suitable to work in a trievents account. On the contrary, the hypervaluations look just what we need, especially because the fact they have "|" makes the null value easy to obtain--given that every $\top|\bot = n$. For such a reason, Mura provided the following account, known as *Semantics of Hypervaluations:*[117]

- Definitions:

  [SH-1]. *Hypervaluation:* given a set S of sentences pertained to $\mathscr{L}$, a hypervaluation associated with a valuation v is the function h_v: $S \to$ {t, f, n} defined recursively by such conditions:

  (1) For every atomic sentence ϕ, $h_v(\phi) = v(\phi)$.

  (2) If $\phi = \sim\psi$ then

  (a) $h_v(\phi)$=t if $h_v(\psi)$=f;

  (b) $h_v(\phi)$=f if $h_v(\psi)$=t;

  (c) $h_v(\phi)$=n if $h_v(\psi)\neq$t and $h_v(\psi)\neq$f.

  (3) If $\phi = (\chi \vee \psi)$ then

  (a) $h_v(\phi)$=t if at least one of the following conditions are satisfied:

  (i) $h_v(\chi)$=t;

  (ii) $h_v(\psi)$=t;

  (iii) for no valuation w, both $h_w(\chi)$ and $h_w(\psi)$ are false, and there is a valuation w' such that either $h_{w'}(\chi)$ or $h_{w'}(\psi)$ are true.

  (b) $h_v(\phi)$=f if at least one of the following conditions are satisfied:

  (i) $h_v(\chi)$=f and $h_v(\psi)$=f;

[117] Alberto Mura, "Probability and the Logic of de Finetti's Trievents".

(ii) for every valuation w, $h_w(\chi) \in \{f, n\}$ and $h_w(\psi) \in \{f, n\}$, and there is a valuation w' such that both $h_{w'}(\chi)$ and $h_{w'}(\psi)$ are false.

(c) $h_v(\phi)$=n if $h_v(\phi)$≠t and $h_v(\phi)$≠f.

(4) If $\phi = (\chi \wedge \psi)$ then

(a) $h_v(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v(\chi)$=t and $h_v(\psi)$=t;

(ii) for every valuation w, $h_w(\chi) \in \{t, n\}$ and $h_w(\psi) \in \{t, n\}$, and there is a valuation w' such that both $h_{w'}(\chi)$ and $h_{w'}(\psi)$ are true.

(b) $h_v(\phi)$=f if at least one of the following conditions are satisfied:

(i) either $h_v(\chi)$=f or $h_v(\psi)$=f;

(ii) for no valuation w, both $h_w(\chi)$ and $h_w(\psi)$ are true, and there is a valuation w' such that either $h_{w'}(\chi)$ and $h_{w'}(\psi)$ are false.

(c) $h_v(\phi)$=n if $h_v(\phi)$≠t and $h_v(\phi)$≠f.

(5) If $\phi = (\chi \rightarrow \psi)$ then $h_v(\phi) = h_v(\sim\chi \vee \psi)$.

(6) If $\phi = (\psi | \chi)$ then

(a) $h_v(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v(\psi)$=t and $h_v(\chi)$=t;

(ii) for every valuation w such that $h_w(\chi)$=t, $h_w(\psi) \in \{t, n\}$, and there is a valuation w' such that both $h_{w'}(\psi)$ and $h_{w'}(\chi)$ are true.

(b) $h_v(\phi)$=f if at least one of the following conditions are satisfied:

(i) $h_v(\psi)$ f and $h_v(\chi)$=t;

(ii) for every valuation w such that $h_w(\chi)$=t, $h_w(\psi) \in \{f, n\}$, and there is a valuation w' such that $h_{w'}(\psi)$=f and $h_{w'}(\chi)$=t.

(c) $h_v(\phi)$=n $h_v(\psi)$≠t and $h_v(\psi)$≠f.

(7) If $\phi = \uparrow\psi$ then

(a) $h_v(\phi)$=t if $h_v(\psi)$=t.

(b) $h_v(\phi)$=f if $h_v(\psi)$=f or $h_v(\psi)$=n.

(8) If $\phi = \updownarrow\psi$

(a) $h_v(\phi)$=t if $h_v(\psi)$=t and $h_v(\psi)$=f.

(b) $h_v(\phi)$=f if $h_v(\psi)$≠t and $h_v(\psi)$≠f.

[SH-2]. *Semantic equivalence:* two sentences ϕ and ψ of $\mathscr{L}$ are semantically equivalent ($\phi \approx \psi$) if and only if, for every valuation v, it holds that $h_v(\phi) = h_v(\psi)$.

[SH-3]. *Pre-tautology:* a sentence ϕ of $\mathscr{L}$ is a pre-tautology if and only if, for every valuation w it holds that $h_w(\phi) \in \{t, n\}$, and there exists a valuation w' such that $h_{w'}(\phi)$=t.

[SH-4]. *Pre-contradiction:* a sentence ϕ of $\mathscr{L}$ is a pre-contradiction if and only if, for every valuation w it holds that $h_w(\phi) \in \{f, n\}$, and there exists a valuation w' such that $h_{w'}(\phi)$=f.

[SH-5]. *Factual sentence:* ϕ of $\mathscr{L}$ is factual if and only if, given two valuations v and w, it holds that $h_v(\phi)$=t and $h_w(\phi)$=f.[118]

[SH-6]. ϕ of $\mathscr{L}$ is a *void* sentence if and only if, for every valuation w, it holds that $h_w(\phi)$=n.

- [SH-THEOREM 1]. For every sentence ϕ, (i) if it is a pre-tautology then ϕ is a tautology--that is for every valuation w, it holds that $h_w(\phi)$=t--and (ii) if it is a pre-contradiction then ϕ is a contradiction--for every valuation w, it holds that $h_w(\psi)$=f.
  - (i). *Proof* (by induction on the construction of ϕ). We shall consider separately the following mutually exclusive cases:
    - If ϕ is an atomic sentence, the thesis is vacuously true because ϕ cannot be either a pre-tautology or a pre-contradiction.
    - If $\phi = \sim\psi$ then ψ is a pre-contradiction and--by inductive hypothesis--ψ is a contradiction, so that for every valuation v it holds that $h_v(\psi)$=f. Hence, ψ is a contradiction and ϕ--according to definition [SH-1] condition (2.a)--is a tautology.
    - If $\phi = (\chi \vee \psi)$ for no valuation w it holds that $h_w(\chi)$=f and $h_w(\psi)$=f, since otherwise $h_w(\phi)$=f, which is inconsistent with [SH-3]. Moreover,

[118] A factual sentence may be true or false. If it can be even null, it does not matter now.

there exists a valuation w' by which $h_{w'}(\phi)$=t so that--by [SH-1] condition (3)--either $h_{w'}(\chi)$=t or $h_{w'}(\psi)$=t or there is a valuation w'' such that either $h_{w''}(\chi)$ or $h_{w''}(\psi)$ are true--implying that $h_{w''}(\phi)$=t. Therefore, for every valuation v it holds that $h_v(\phi)$=t, so that ϕ is a tautology.

- If $\phi=(\chi\wedge\psi)$ then both χ and ψ are pre-tautologies and--by inductive hypothesis--χ and ψ are tautologies. Then, for every valuation w, $h_w(\chi)$=t and $h_w(\psi)$=t. Consequently, for every valuation v it holds that $h_v(\phi)$=t, so that ϕ is a tautology.
- If $\phi=(\chi\rightarrow\psi)$ then for every valuation v it holds that $h_v(\phi)=h_v(\sim\chi\vee\psi)$--let $\chi'=\sim\chi$, so that $\phi=(\chi'\vee\psi)$. The theorem holds in virtue of what has been said about the case in which $\phi=(\chi\vee\psi)$.
- If $\phi=(\psi|\chi)$ then $\chi\rightarrow\psi$ is a pre-tautology, so that ϕ is a tautology--as shown.
- If $\phi=\uparrow\psi$ then ψ is a tautology, so that for every valuation v it holds $h_v(\phi)$=t--by [SH-1] condition (7.a.)--and ϕ is a tautology.
- If $\phi=\updownarrow\psi$ then for every valuation v it holds $h_v(\phi)\in$ {t, f}--by [SH-1] condition (8.a.). But, for hypothesis, ϕ is a pre-tautology so that it cannot be $h_v(\phi)$=f. It follows that $h_v(\phi)$=t, for every valuation v. Hence, ϕ is a tautology.

- (ii). *Proof.* If ϕ is a pre-contradiction then $\sim\phi$ is a pre-tautology and, by (a), a tautology. Now, for every valuation v it holds that $h_v(\phi)=h_v(\sim\sim\phi)$--[SH-1] condition (2). Since $h_v(\sim\phi)$=t, it holds that $h_v(\sim\sim\phi)$= f = $h_v(\phi)$--[SH-1] condition (2). Hence, ϕ is a contradiction.

In such a way Mura provided a semantics able to remove every pre-tautology and pre-contradiction, compositionally converting them respectively in standard tautologies and standard contradictions.

However, hypervaluations' account maintains some difference from classical logic:

- *No unrestricted substitution rule* holds for the Semantics of Hypervaluations. It means that every schema of tautologies and contradictions does not represent a class of valid sentences as well as the same schema represents a class of quasi-tautologies in a three-valued logic. That because not every instance of the schema is a tautology. So, we will have troubles in considering sentences like $\phi \wedge \sim\phi$ as tautologies. Indeed, $\phi \wedge \sim\phi$ is valid if and only if ϕ is not null. However, to treat a "$\phi \wedge \sim\phi$" schema as a standard one we can fix the condition "If ϕ is not null, then $\phi \wedge \sim\phi$ is a tautology". Generally, it is not immediately decidable that ϕ is not null, because it does not depend just by inspecting ϕ. But it is possible to identify algorithmically a void sentence[119].
- In the Semantics of Hypervaluations *the truth conditions of sentences are not given by simple truth tables*. That because [SH-1] entails a modal component, due to a reference to the set of all valuations. However, the following theorem shows that the truth-value of a sentence depends on the valuation of the atomic sentences occurring in it:

- [SH-THEOREM 2]. Be ϕ be a sentence of $\mathscr{L}$ and $\psi_1,\ldots,\psi_2$ the atomic sentences occurring in ϕ. If there are two valuations v and v' such that for every i--with $1 \leq i \leq n$--it holds that $v(\psi_i) = v'(\psi_i)$, then $h_v(\phi) = h_{v'}(\phi)$.
  - *Proof.* By induction on a number n of connectives occurring in ϕ, if $n = 0$, then ϕ is an atomic sentence. So, it holds that $h_v(\phi) = v(\phi) = v'(\phi) = h_{v'}(\phi)$--by [SH-1], according the condition for which, for every atomic sentence ϕ, $h_v(\phi) = v(\phi)$. Supposing that $n = m + 1$ and that, for every $k \leq m$, [SH-THEOREM 2] is true, we have these cases:
    - Representing any unary connective with "$\oplus$", for $\phi = \oplus\psi$ it holds that $h_v(\psi) = h_{v'}(\psi)$--by inductive hypothesis--, so that $h_v(\phi) = h_{v'}(\phi)$--by [SH-1], according the conditions for $\sim$, $\uparrow$ and $\updownarrow$.

[119] The problem about considering $\varphi \wedge \sim\varphi$ such a tautology is solved in Mura 2012. See Alberto Mura, "Towards a New Logic of Indicative Conditionals", *Logic and Philosophy of Science*, vol. IX, No. 1 (2011): 17--31.

- Representing any binary connectives with "⓪", for $\phi = \chi ⓪ \psi$ it holds that $h_V(\chi) = h_{V'}(\chi)$ and $h_V(\psi) = h_{V'}(\psi)$--by inductive hypothesis--, so that $h_V(\phi) = h_{V'}(\phi)$ by [SH-1], according the conditions for ∨, ∧, → and |.

[SH-THEOREM 2] shows that, in spite of its modal component, the hypervaluation's account may be considered truth-functional because any connective of $\mathscr{L}$ is truth-functional. However, it is not truth-functional in a strict (classical) meaning[120], but in a generalized sense. In other words, it respects such a *general definition of truth-functionality:*

- [SH-THEOREM 3]. Any sentential (*n*-ary) connective ⊛ is truth-functionally in a generalized sense if and only if the truth-value of any sentences $\psi = ⊛(\phi_1,...,\phi_n)$ is a function of the truth-values of the atomic sentences $p_1, ..., p_n$ occurring on ψ.

According to [SH-THEOREM 3], the Semantics of Hypervaluations allows to obtain a truth-table for a molecular sentence determined by every truth-value of the atomic sentences occurring in it. Mura 2009 called this procedure "mutant truth-tables". It simply consists "in developing the original truth-table algorithm, every computed column is checked for pre-tautology or pre-contradiction and it is immediately conserved into a tautology or contradiction respectively *before the process continues*".[121] In other words, Mura suggested a three-step procedure:

(Step 1). Build the original de Finetti's truth-table for a molecular sentence.

(Step 2). Assume every quasi-tautology--a trievent true or null, but never false-- and every quasi-contradiction--a trievent false or null, but never true-- as a pre-tautology and a pre-contradiction respectively. In such a way we obtain a new (mutant) truth-table in which all quasi-tautologies and

[120] Any sentential (*n*-ary) connective ⊛ is truth-functionally in a classical sense if the truth-value of any sentence $⊛(\varphi_1,...,\varphi_n)$ is a function of the truth-values of the sentences $\varphi_1,...,\varphi_n$.

[121] Alberto Mura, "Probability and the Logic of de Finetti's Trievents", 228.

quasi-contradictions are immediately converted in tautologies or contradictions.

(Step 3). Check the truth-value of the molecular sentence at the light of this mutant truth-table.

Here an example: consider the molecular sentence *(p|p)* ∧ *(~p|~p)*.

(Step 1). Original de Finetti's truth-table:

p	*~p*	***p|p***	***~p|~q***	*(p|p)∧(~p|~p)*
T	F	**T**	**N**	N
F	T	**N**	**T**	N
N	N	**N**	**N**	N

Quasi-tautologies

(Step 2). Mutant truth-table:

p	*~p*	***p|p***	***~p|~q***	*(p|p)∧(~p|~p)*
T	F	**T**	~~N~~ **T**	~~N~~ T
F	T	~~N~~ **T**	**T**	~~N~~ T
N	N	~~N~~ **T**	~~N~~ **T**	~~N~~ T

Pre-tautologies = Tautologies

(Step 3). The sentence *(p|p)* ∧ *(~p|~p)* is now a tautology:

p	$\sim p$	$p \mid p$	$\sim p \mid \sim q$	$(p \mid p) \wedge (\sim p \mid \sim p)$
T	F	T	T	**T**
F	T	T	T	**T**
N	N	T	T	**T**

In conclusion, the mutant procedure allows to solve the problem of original de Finetti's semantic related to the fact that $\phi \mid \phi$ is not a tautology. Such a limit leads to consider sentences like "Supposed that it is raining then it is raining, and supposed that it is not raining then it is not raining" as null. Instead, the Semantics of Hypervaluations makes us able to treat such sentences as tautologies, in perfect accordance with the common use in natural language.

Although the Semantics of Hypervaluations can solve some problems of de Finetti's trievents, it is not enough to overcome every limit the original account presents. For example, it still cannot avoid McGee's result, so that an incompatibility with Adams' account keeps on holding. For this reason, Mura 2012 proposed a refinement of the previous Semantics of Hypervaluations, with the intention of figure that out.

2. Theory of Hypervaluated Trievents

The refined new account proposed by Mura 2012 is known as "Theory of Hypervaluated Trievents".[122] It basically consists in defining a hypervaluation, not just in respect of a single valuation, but of a *set* of valuations. So, we have such a definition:

> [THT-1]. *Hypervaluation:* given a set S of sentences pertained to $\mathscr{L}$, a hypervaluation associated with a valuation v and with a set V of valuations

[122] Alberto Mura, "Towards a New Logic of Indicative Conditionals", *Logic and Philosophy of Science*, vol. IX, No. 1 (2011): 17--31.

(such that $v \in V$) is the function $h_v^V: S \rightarrow$ {t, f, n} defined recursively by such conditions:

(1) For every atomic sentence ϕ, $h_v^V(\phi) = v(\phi)$.

(2) If $\phi = {\sim}\psi$ then

(a) $h_v^V(\phi)$=t if $h_v^V(\psi)$=f;

(b) $h_v^V(\phi)$=f if $h_v^V(\psi)$=v;

(c) $h_v^V(\phi)$=n if $h_v^V(\psi)$≠t and $h_v^V(\psi)$≠f.

(3) If $\phi = (\chi \vee \psi)$ then

(a) $h_v^V(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v^V(\chi)$=t;

(ii) $h_v^V(\psi)$=t;

(iii) for no valuation $w \in V$, both $h_w^V(\chi)$ and $h_w^V(\psi)$ are false, and there is a valuation $w' \in V$ such that either $h_{w'}^V(\chi)$ or $h_{w'}^V(\psi)$ are true.

(b) $h_v^V(\phi)$=f if at least one of the following conditions are satisfied:

(i) $h_v^V(\chi)$=f and $h_v^V(\psi)$=f;

(ii) for every valuation $w \in V$, $h_w^V(\chi) \in$ {f, n} and $h_w^V(\psi) \in$ {f, n}, and there is a valuation w' such that both $h_{w'}^V(\chi)$ and $h_{w'}^V(\psi)$ are false.

(c) $h_v^V(\phi)$=n if $h_v^V(\phi)$≠t and $h_v^V(\phi)$≠f.

(4) If $\phi = (\chi \wedge \psi)$ then

(a) $h_v^V(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v^V(\chi)$=t and $h_v^V(\psi)$=t;

(ii) for every valuation $w \in V$, $h_w^V(\chi) \in$ {t, n} and $h_w^V(\psi) \in$ {t, n}, and there is a valuation w' such that both $h_{w'}^V(\chi)$ and $h_{w'}^V(\psi)$ are true.

(b) $h_v^V(\phi)$=f if at least one of the following conditions are satisfied:

(i) either $h_v^V(\chi)$=f or $h_v^V(\psi)$=f;

(ii) for no valuation w, both $h_w(\chi)$ and $h_w(\psi)$ are true, and there is a valuation w' such that either $h_{w'}(\chi)$ and $h_{w'}(\psi)$ are false.

(c) $h_v^V(\phi)$=n if $h_v^V(\phi)$≠t and $h_v^V(\phi)$≠f.

(5) If $\phi = (\chi \rightarrow \psi)$ then

(a) $h_v^V(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v^V(\psi)$=t;

(ii) $h_v^V(\chi)$=f;

(iii) $h_v^V(\chi)$=n and $h_v^V(\psi) \in$ {t, n}.

(b) $h_v^V(\phi)$=f otherwise.

(6) If $\phi = (\psi|\chi)$ then

(a) $h_v^V(\phi)$=t if at least one of the following conditions are satisfied:

(i) $h_v^V(\psi)$=t and $h_v^V(\chi)$=t;

(ii) for every valuation $w \in V$ such that $h_w^V(\chi)$=t, $h_w^V(\psi) \in$ {t, n}, and there is a valuation w' such that both $h_{w'}^V(\psi)$ and $h_{w'}^V(\chi)$ are true.

(b) $h_v(\phi)$=f if at least one of the following conditions are satisfied:

(i) $h_v^V(\psi)$ f and $h_v^V(\chi)$=t;

(ii) for every valuation w such that $h_w^V(\chi)$=t, $h_w^V(\psi) \in$ {f, n}, and there is a valuation w' such that $h_{w'}^V(\psi)$=f and $h_{w'}^V(\chi)$=t.

(c) $h_v^V(\phi)$=n otherwise.

(7) If $\phi = \uparrow\psi$ then

(a) $h_v^V(\phi)$=t if $h_v^V(\psi)$=t.

(b) $h_v^V(\phi)$=f otherwise.

(8) If $\phi = \updownarrow\psi$

(a) $h_v^V(\phi)$=t if either $h_v^V(\psi)$=t or $h_v^V(\psi)$=f.

(b) $h_v^V(\phi)$=f otherwise.

(9) If $\phi = \top$ then $h_v^V(\phi)$=t.

(10) If $\phi = \bot$ then $h_v^V(\phi)$=f.

(11) If $\phi = \mathbb{N}$ then $h_v^V(\phi)$=n.

In addition, the Theory of Hypervaluated Trievents provides a definition of modal connectives in terms of primitive connectives:

[THT-2]. *Modal operators:*

- ϕ is void = $\boxtimes\phi \stackrel{\text{def}}{=} \sim\uparrow(\phi \vee \sim\phi)$
- ϕ is possibly true = $\blacklozenge\phi \stackrel{\text{def}}{=} (\top \rightarrow (\phi|\phi))$
- ϕ is possible = $\lozenge\phi \stackrel{\text{def}}{=} \blacklozenge\phi | \sim\boxtimes\phi$

- ϕ is necessary = $\Box\phi \overset{\text{def}}{=} \sim\Diamond\sim\phi$
- ϕ is necessarily true = $\blacksquare\phi \overset{\text{def}}{=} \Box\uparrow\phi$

Concerning any other notion, of course it must also be defined in relation to a set of valuations:

[THT-3]. *Validity:* ϕ is a valid sentence if and only if, for every non-empty set V of valuations, ϕ is valid with respect to V.

- The rule of substitution is restored--by [THT-1]--, so that the previous problem holding in the Semantics of Hypervaluations, concerning to consider general sentences like $\phi\wedge\sim\phi$ as tautologies, is solved. Therefore, now it is possible to use *valid schemas* to represent classes of valid sentences.

[THT-4]. *Logical equivalence:* ϕ and ψ are logically equivalent if and only if, for every V and every $v\in V$, it holds that $h_v^V(\phi) = h_v^V(\psi)$. It follows (i) that ϕ and ψ are logically equivalent if and only if $\phi\leftrightarrow\psi$ is valid--by [THT-1], and (ii) the theorem below:

- [THT-THEOREM 1] Every sentence ϕ of $\mathscr{L}$ is logically equivalent to a sentence ψ of the form $\phi'|\phi''$ such that the connective "|" does not occur neither in ϕ' nor in ϕ'' and that every atomic sentence of both ϕ' and ϕ'' is immediately preceded by "↑" or "↑∼".

[THT-5]. *Probability:* given a set V of valuation and a valuation $v\in V$, the hypervaluation $h_v^V(\phi)$ represents an extreme probability function assigning probability 1 and 0 to every true and every false trievent respectively, and remaining undefined when a trievent is neither true nor false.

- Given that a trievent X may represent a simple conditional--although it is not a standard proposition--, its probability $\boldsymbol{P}$ can be interpreted as the expectation $\boldsymbol{E}$ of its truth-value conditional on the hypothesis that the trievent is either true or false:

  $\boldsymbol{P}(X) = \frac{\boldsymbol{E}(X)}{\boldsymbol{E}(X=0 \ or \ X=1)} = \boldsymbol{P}\frac{\uparrow X}{\updownarrow X}$, with $\boldsymbol{P}(\updownarrow X) > 0$.

- Given a trievent expressing the simple conditional $q\,|\,p$, its probability can be so defined--avoiding Lewis' Triviality Result--:

  $P(q\,|\,p) = P\frac{\uparrow(q|p)}{\updownarrow(q|p)} = \frac{\boldsymbol{P}(p\wedge q)}{\boldsymbol{P}(p)}$.

Now, to provide a semantics for Adams' logic by a three-valued account it is not enough to prove that a trivialization can be avoid. Indeed, if the McGee's result still keep on holding then no many-valued logic can preserve Adams' definition of logical consequence. For that reason, Mura suggested a notion of logical consequence modified on a set of hypervaluations.

3. Logical consequence

Providing a definition of logical consequence in the Theory of Hypervaluated Trievents able to get over McGee's result, Mura's intent is to build a semantic apparatus for Adams' conditional logic and to extend it to all Trievents--including compound conditionals.

Such a notion of logical consequence is so defined:

[THT-6] *Logical consequence:* ψ is a logical consequence of ϕ if and only if, for every set V of Hypervaluations, there is no $v \in V$ such that $h_v^V(\phi)$ is true but $h_v^V(\psi)$ is not true, and there is $v \in V$ such that $h_v^V(\phi) \in$ {t, n}, i.e. $\phi \vDash \psi$ if and only if, for every set V, both preservation of truth and preservation of non-falsehood are respected.

This explains why the connective "→" adopted in the Theory of Hypervaluations represents the material implication.[123] Indeed, it is generally required for a material implication the property according to which $\phi \rightarrow \psi$ is valid if and only if ψ is a *logical consequence* of ϕ--in a [THT-6] sense. Given that it is exactly the semantic fixed by [THT-1] for "→", the connective expressed by "→" in [THT-1] is just the material implication for [THT-6]. No other binary connectives can satisfy that property.

[123] The fact that in such a theory there is the material conditional is important even in case of *modus ponens*. Indeed, this is a principle holding just with simple conditionals, but not with compounds. However, it is not a surprise given that the connective "|" is not the material implication. In other words, it does not matter if the modus ponens does not work with "|", because in Mura's account there is a "→" that is material.

Now, it should be pointed out that no every property of the standard logical consequence keeps on holding in the Theory of Hypervaluated Trievents. Indeed, it is not preserved the classical property according to which, given a finite set of sentences $K=\{\phi_1, \ldots, \phi_n\}$, ψ is a logical consequence of K if and only if ψ is a logical consequence of $\phi_1 \wedge \ldots \wedge \phi_n$. That because, as previously anticipated, the *introduction rule for the conjunction is not valid* in this logic. It means that, for every i, while ϕ_i is a logical consequence of $\phi_1 \wedge \ldots \wedge \phi_n$, $\phi_1 \wedge \ldots \wedge \phi_n$ is not a logical consequence of K.

The lack of the introduction rule makes that $\{\phi, \psi\}$ does not entail in an Adams' sense--or "p-entail"--the conjunction $\phi \wedge \psi$. So, pragmatically speaking, asserting all together two or more sentences does not equal asserting their conjunction. That sounds weird because we are accustomed to think the simultaneous assertion of "If it rains, I'll stay at home" and "If it does not rain, I'll go to the beach" exactly as the assertion of "If it rains, I'll stay at home and if it does not rain, I'll go to the beach". But, according to a trievents account such a conjunction would be semantically null. Therefore, we should not interpret "and" as a connective between conditional sentences, but rather as *a connective between speech acts*.

However, we may adopt a three-valued conjunction for which the introduction rule holds. Such a different connective has been introduced by Adams himself--but only for simple conditionals. It is called "quasi-conjunction" and it "is verified if and only if none of its parts are falsified and at least one is verified". [124] In a three-valued logic, that should be interpreted in such a way that a null conjunct is futile in determining the truth-value of the conjunction--unless both conjuncts are null, so that the conjunction is null as well. Hence, this is the truth-table for a quasi-conjunction "$\owedge$":

[124] Ernest W. Adams, *A Primer of Probability Logic* (Stanford: Csli Publications, 1998), 172.

ϕ	ψ	$\phi \wedge\!\!\!\wedge \psi$
T	**T**	T
T	**F**	F
T	**N**	T
F	**T**	F
F	**F**	F
F	**N**	F
N	**T**	T
N	**F**	F
N	**N**	N

We can now define "⩕" according to the Semantics of Hypervaluations, in terms of primitive connectives:

$\phi \wedge\!\!\!\wedge \psi \overset{\text{def}}{=} (\sim\uparrow\sim\phi \wedge \sim\uparrow\sim\psi) \mid (\updownarrow\phi \vee \updownarrow\psi)$.

In such a way, we have a connective for which the introduction rule is valid. But, on the other hand, we lose the elimination rule for quasi-conjunction--according to which: $\phi \wedge\!\!\!\wedge \psi \vdash \phi$ and $\phi \wedge\!\!\!\wedge \psi \vdash \psi$. But this is not something new. Indeed, Adams had already shown that there is no formula of our language such that both introduction and elimination rules for conjunction are in accordance with the p-entailment.[125]

However, although the limit concerning the elimination rule, preserving the introduction rule is now possible to give a general definition of logical consequence from a finite set of sentences:

[THT-7]. *Generalization of logical consequence:* for every finite set of sentences $\Gamma = \{\phi_1, .., \phi_n\}$--with $1 \leq n \leq w$--, $\Gamma \vDash \psi$ if and only if either ψ is valid or there is a subset $\Gamma' = \{\phi_{i1}, .., \phi_{ik}\}$ of Γ--with $k \leq n$--such that $\{\phi_{i1} \wedge\!\!\!\wedge \dots \wedge\!\!\!\wedge \phi_{ik}\} \vDash \psi$.

[125] Ernest W. Adams, *A Primer of Probability Logic*, 177.

In such a way the quasi-conjunction allows a semantical generalization of Adams' p-entailment, so questioning the idea that conditionals always lack of truth conditions. This is proved by the following theorem:

- [THT-THEOREM 2]. Given a finite set $\Gamma=\{\phi_1, .., \phi_n\}$ of non-void sentences of $\mathscr{L}$ and a non-void sentence ψ of $\mathscr{L}$, the following propositions are equivalent:
    - For every probability function P defined for every element of Γ and for ψ, it holds that $P(1-\psi) \leq \sum_{i=1}^{n}(1-P(\varphi_i))$.
    - $\Gamma \vDash \psi$.
  - *Proof.* Such a theorem can be proved easily by indirect way:
    - By de Finetti's Decomposition Theorem, every conditional is simple and it must have the same probability of a simple conditional.
    - Every axiom of Adams' logic is satisfied for simple conditionals.
    - Even Adams' p-entailment is extended.

In conclusion, [THT-THEOREM 2] shows that [THT-7] includes Adams' p-entailment and allows an extension of it for all trievents in the Semantics of Hypervaluations. In this way, Adams' logic can be interpreted as a fragment of a partial modal (three-valued) logic, and we shall have a useful tool for dealing with compound sentences.[126]

4. Trievents and counterfactuals

Although Mura's account can get over some limits of the original de Finetti's trievents, it does not allow prima facie to deal with counterfactuals. Indeed, in both accounts it holds that, when the probability of the antecedent is 0, then the conditional is null--it is neither true nor false. Therefore, every counterfactual seems

[126] A fragment of such a hypervaluated trievents account can be considered as a three valued version of the S5 system. That because, for every hypervaluated trievent φ it exists a corresponding S5-formula φ' such that φ' is S5-valid if and only if φ is valid in the Theory of Hypervaluated Trievents. However, that is still a work in progress.

to be semantically and pragmatically void. This is a limit holding even for Adams' logic, so that he restricted his theory to those conditionals whose antecedent is not known for certainty to be true or false--that are indicative conditionals.

Now, an account able to provide a unified thesis for both indicative and counterfactual conditionals definitely conserves an advantage. We have already seen that Stalnaker's theory can work--although not free from problems--with both conditionals, but it has to pay the prize to give up the Equation. The trievents account can represent a good option for those who do not want to reject such an important intuitive result, but for competing with any other unified theory it is important an extension to counterfactuals. For this reason, Mura proposed a generalization of the Theory of Hyervaluated Trievents able to catch counterfactual conditionals.[127] It may be possible introducing a new variable *K* representing the set of accepted ordinary propositions--that are propositions considered true until a contrary new evidence. Every trievent is always related to a set *K* of total beliefs.

The basic idea is that every information has to communicate something new, other from those beliefs two (or more) people have already in common. This is absolutely plausible for a pragmatic point of view, and it can be semantically respected either. Indeed, such a "something new" is simply the *epistemic content* of a proposition, keeping out *K* and representing just what a proposition means.

So, a generalization of the Theory of Hypervaluated Trievents is given representing any trievent $(q\,|\,p)$ such as $(q\,|\,p : K)$, to indicate that it is always associated to a stock *K* of beliefs. In this way every conditionals is not *essentially* indicative or counterfactual, but just in relation to *K*. Indeed, in case of indicative conditionals, *K* will be the corpus of the *actual* beliefs people have at that moment. Instead, a counterfactual is not related to an actual *K*, so that it would be wrong to consider it simply as an indicative conditional whose antecedent *p* is false--being so null. Indeed, if our actual corpus of beliefs is *K*, asserting a counterfactual we should

[127] Alberto Mura, "Ragionamento probatorio a partire da premesse incerte e asserti condizionali", *Prospettive Interdisciplinari Per La Giustizia Penale* (2014): forthcoming in *Cassazione Penale*.

suppose a different *hypothetic* stock *K'*, containing the same information of *K* but differing only about the antecedent--$\sim p$ is now removed and replaced with *p*--and everything correlated with it.

Thus, for example, the indicative conditional “If Oswald didn’t kill Kennedy, someone else did” is related to a stock *K* containing the information that Kennedy was killed, but nothing about Oswald such as murderer. Instead, the respective counterfactual “If Oswald hadn’t killed Kennedy, someone else would have killed him” is related to a corpus *K'* containing the information that Oswald killed Kennedy and nothing about the consequent’s truth-value.

In such a way, the truth conditions of conditional sentences are fixed in relation to our set of beliefs. In case of counterfactuals, their truth-value equals that one of the consequent *q*, on supposition of a hypothetic set of information. When *q* is a standard bivalent proposition, the counterfactual will express a bivalent proposition too, because its antecedent *p* is entailed by *K'*. It means that, while an indicative can be null, a simple counterfactual is always either true or false--but never null.

Similarly, a generalization of the Ramsey’s test is given considering $\boldsymbol{P}(q \mid p)$ in relation to a set *K* of knowledge or suppositions. So we can reformulate the Equation in such a way:

$$\boldsymbol{P}(p \rightarrow q : K) = \boldsymbol{P}(q \mid p : K).$$

Therefore, the difference between the two classes of conditionals is not essentially logical, but *epistemic*--the logic of the conditionals is the same either for indicatives, either for counterfactuals. So, logically speaking there is just one conditional. The difference between indicatives and counterfactuals is given by the pragmatic fact that using the subjunctive tense there is a conventional implication--the speaker makes to understand that it knows about antecedent’s falsity and that its stock of knowledge is currently different.

If Mura’s suggestion, which is still a work in progress, holds then we can have a unified theory allowing to treat indifferently either indicative either counterfactual conditionals. But, the crucial point is to define the passage from *K* to *K'*.

Basically, Mura advances a three-step propose--although he is still working to better define it:

(Step 1). Take the whole Boolean algebra $\mathscr{B}$ generated by *K*.

(Step 2). Remove the proposition *p* from $\mathscr{B}$ and everything not separable from *p* in $\mathscr{B}$, generating the corpus K^{-} of remaining information of $\mathscr{B}$. The notion of *separability* is the following:

- Given a Boolean algebra[128] $\mathscr{B}$ and two elements *p* and *q* in $\mathscr{B}$--with p, $q \neq 0$--, *p* and *q* are said logically separable in $\mathscr{B}$ if and only if there are two independent sub-algebras $\mathscr{B}'$ and $\mathscr{B}''$ of $\mathscr{B}$ such that (i) *p* belongs to $\mathscr{B}'$, (ii) *q* belongs to $\mathscr{B}''$, and (iii) $\mathscr{B}' \cup \mathscr{B}''$ generate $\mathscr{B}$.[129]

(Step 3). Add $\sim p$ to K^{-}, generating a new corpus *K'* of information.

Now, either *K* or *K'* generates a Boolean algebra and, technically, a class of (logically) possible worlds. Therefore, the problem to move from *K* to *K'* is essentially the same (concerning the formal logic) of finding a possible world closest to the actual one but differing about the antecedent and everything linked to it--and nothing more. Stalnaker himself introduced some restriction for a set of worlds based on some given or contextual information, so that there is no such a logical difference between that set and a stock *K* of knowledge. The difference concerns a metaphysical level.

However, the concept of "possible world" was introduced first by Leibniz.[130] Indeed, even if in an informally way, he anticipated the basic idea of Stalnaker's account about conditionals. In *Théodicée* Leibniz used the following words:

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[128] The idea to move in a Boolean algebra scenario due to the fact that here two equivalent languages correspond to the same (Boolean) algebra. In this way no accidental aspect of the language is introduced, and the famous "grue paradox" of Goodman does not hold.

[129] Mura introduced such a notion of logical separability for the first time in: Alberto Mura, "When Probabilistic Support is Inductive", *Philosophy of Science*, vol. 57, No. 2 (The University of Chicago Press: 1990), 278--289.

[130] In a personal conversation Stalnaker told me that he did not have in mind Leibniz's concept of possible world. In addition, according to Stalnaker, Leibniz gave just a superficial definition, without

"[…] Car le cas du siége de Kégila est d'un monde possible qui ne diffère du nôtre qu'en tout ce qui a liaison avec cette hypothèse, et l'idée de ce monde possible représente ce qui arriverait en ce cas. Donc nous avons un principe de la science certaine des contingents futurs, soit qu'ils arrivent actuellement, soit qu'ils doivent arriver dans un certain cas."[131]

Leibniz limited to talk about a "liaison", that literally means "link". The *relation of similarity* is a notion due to Stalnaker-Lewis' semantics. They interpreted a possible world which differs only for all that is related with the hypothesis such as *the most similar* world to the actual one.[132] But, actually, Leibniz has never talked in terms of similarity. Now, according to Mura, what Leibniz had in mind was, rather, a *ceteris paribus condition* which should not be interpreted in terms of similarity between worlds *á la* Stalnaker. Basically, the idea is:

"When we consider whether a counterfactual A>B is true, we imagine a ceteris paribus A-world, one where A holds and other things remain "equal." However, the ceteris paribus condition should not be confused with similarity between worlds. When we consider (4)[If President Kennedy had pushed the red button during the Cuban missile crisis, then there would have been a nuclear Holocaust], for example, we hold fixed the causal regularities tying the red button to a missile launch, and this means we consider devastated worlds that are radically unlike our own. Exactly how ceteris paribus".[133]

But how to interpret such a ceteris paribus condition? Mura suggests to explain it just in terms of *separability*, such as reported exactly in the second step defining the passage from *K* to *K'*. In this way it would be possible avoid a notion of similarity

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really develop such an issue. However, Leibniz was the first who mentioned the idea of possible world, so that I think it would be interesting get a look at his idea.

[131] Gottfried W. Leibniz and Louis de Jaucourt, *Essais de Théodicée sur la bonté de Dieu, la liberté de l'homme et l'origine du mal*, Vol. 2 (chez François Changuion, 1747), 135. Translated in English as: "For the case of the siege of Keilah forms part of a possible world, *which differs from ours only in all that is connected with this hypothesis*, and the idea of this possible world represents that which would happen in this case. Thus we have a principle for the certain knowledge of contingent futurities, whether they happen actually or must happen in a certain case." Gottfried W. Leibniz, *Theodicy*, ed. Austin M. Farrer (New York: Cosimo, Inc., 2009), 146.

[132] Or the most similar *worlds*, according to Lewis.

[133] James W. Garson, *Modal logic for philosophers* (Cambridge: Cambridge University Press, 2013), 460--461.

and all those troubles related to it--like that one showed by the example of the nuclear Holocaust.[134]

5. Compound conditionals

An important advantage given by Mura's account is that every trievent is a *simple event*. Indeed, the Theory of Hypervaluated Trievents allows, by de Finetti's Decomposition Theorem, to simplify every trievent so that each compound conditional can be analyzed such as a simple one. In this way, we will be able to extend Adams' logic to compound conditionals.

I think this is a great result because, although compound conditionals are complicated sentences, it is unquestionably we often use some of them in everyday reasoning. So, I consider quite inappropriate Adams' suggestion to give up compound conditionals considering them not tractable at all.

For example, a common compound construction is given by the import-export. Therefore, it would be great that a logic of conditionals could preserve such an important logical principle. Now, while imp-exp does not hold in Stalnaker's account, in the Theory of Hypervaluated Trievents it is generally confirmed:[135]

- for every ordinary proposition p, q and z, we have the following necessary and *all together* sufficient conditions:[136]
  - $(\Box \uparrow (q \mid p) \wedge \Diamond z \wedge \sim\Diamond(p \wedge z)) \rightarrow \Box q$.
  - $(\Box q \vee \Box\sim q) \rightarrow [\Diamond(\mathrm{p} \wedge \mathrm{z}) \vee \sim \Delta\, p \vee \sim \Delta\, z]$.
  - $((\Box q \vee \Box\sim q) \wedge \Delta\, p \wedge \Delta\, z) \rightarrow \Diamond(p \wedge z)$.

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[134] Troubles that Lewis himself pointed out at the moment to choose just one world (as Stalnaker's Uniqueness Assumption intends).

[135] Except for a few cases. For example, imp-exp is not generally confirmed with sentences of the form $p \rightarrow (q \rightarrow p)$. Indeed, when p and q cannot be both true we have sentences like "If Harry runs fifteen miles this afternoon, then if he is killed in a swimming accident this morning, he will run fifteen miles this afternoon". This does not seem a logical truth, but just a paradoxical situation. See William G. Lycan, *Real Conditionals* (Oxford: Oxford University Press, 2001), 82--proposing a Gibbard's example. However, imp-exp is *always* confirmed in de Finetti's (original) trievents.

[136] Where $\triangle$ means "factual", i.e. the proposition can be true or false.

- $\big((\Box q \vee \Box{\sim}q) \wedge {\sim}\Diamond(p \wedge z)\big) \rightarrow ({\sim}\Diamond p \vee {\sim}\Diamond z)$.
- $[(\Box \uparrow (\, q \mid p \,) \vee \Box \uparrow {\sim}(\, q \mid p \,) \wedge \Diamond z] \rightarrow \Diamond(p \wedge z)$.

Another important logical principle, holding either in Stalnaker's either in Mura's, is the conditional excluded middle--$(p > q) \vee (p > {\sim}q)$. According to CEM, it must be impossible that either $p > q$ either $p > {\sim}q$ are both true. But, as previously said, Lewis points out that conditionals like "If Bizet and Verdi were compatriots, Bizet would be Italian" and "If Bizet and Verdi were compatriots, Bizet would not be Italian" seem both acceptable.[137] In addition, he also complains CEM might help to choose the closest world to the actual one. Indeed, is it a world where, if Bizet and Verdi were compatriots, then Bizet would be Italian or where Verdi would be French?[138]

Stalnaker supported CEM by Van Fraassen's notion of "vagueness", which allows there might be cases where neither $p \rightarrow q$ nor $p \rightarrow {\sim}q$ are true. This holds for trievents too, because both may be null. In that case, Mura's account works *(step 1)* considering the whole Boolean algebra generated by the actual status K where Bizet and Verdi are not compatriots, *(step 2)* eliminating such evidence and everything which is not separable from it, *(step 3)* adding the information that Bizet and Verdi are compatriots, generating so a new corpus K'. Finally, if and only if the consequent is true in K', the conditional is true too.

Now, we cannot believe that a same proposition is either true either false, so that I cannot assume contradictory information into a same corpus of knowledge. This means, supposing Bizet and Verdi are compatriots, if I assume that Bizet would be Italian I cannot think also that he is not. That is, basically, what Stalnaker proposed by Van Fraassen's.

[137] See note 64.

[138] See note 63.

CONCLUSION

Adams provided an account supporting the Equation, according to which the probability of a conditional is its conditional probability. This is an important intuitive result, and Edgington presented strong arguments in its support. In addition, in a first moment, Stalnaker's theory coincided with Adams' proposal showing that the Equation is generally satisfied with simple conditionals.

Unfortunately, Lewis' Triviality Result showed *prima facie* the incompatibility between the assumption that the probability of a proposition is the probability it is true and the Equation. Consequently, supporting Stalnaker's semantic means to reject Adams' logic--and vice versa.

In front of the Triviality Result, Stalnaker finally gave up the Equation such as a general satisfied principle and suggested to consider conditional sentences as standard propositions. Instead, Adams concluded that conditionals do not have truth conditions of any kind, developing a non-propositional view. Indeed, he thought that Lewis' result holds just because we make the mistake to assign a truth-value to conditionals, while such sentences are never true or false but simply probable or improbable.

Now, although Adams analyzed conditional sentences in terms of probability--rather than truth-values--since 1965, he *explicitly* denied they have any kind of truth conditions only after Triviality Result had been formulated. But, saying conditionals have a probability that is a conditional probability and holding they never own a truth-value mean two different things. Unquestionably, if we have in mind material conditional's truth conditions--interpreting "→" such as "⊃"--, we will meet a lot of troubles. However, that simply means "→" cannot have the same truth conditions of "⊃". Adams' idea that conditionals do not have *general* truth conditions emerged just in front of Lewis' trivialization. For this reason, I have some doubts about Adams' conclusion if no Triviality Result had been advanced. I want to say that to deny conditionals have truth conditions does not look an essential property of conditional sentences, but just an idea developed in front of Lewis' trivialization. In other word, finding a way to avoid such a result might lead to a different conclusion.

In addition, I judge quite "extreme" Adams' suggestion because it is not complicate to find in natural language several examples of conditional sentences considered true or false by our common sense. Edgington herself admitted--during a seminar at University of Sassari--that *sometimes* a conditional could have a truth-value. However, although she looked less radical than Adams, she refused to talk in terms of *general* truth conditions. After all, the fact that we are not always able to assign a truth-value is unquestionable. Even Stalnaker had to admit--especially after Lewis' objections--that sometimes we cannot choose the possible world closest to the actual one and, consequently, the conditional is neither true nor false--reason for what he dealt with Van Fraassen's supervaluations.

Therefore, it seems we need a kind of middle way able to fix a truth-value when possible and considering null those conditionals we cannot judge neither true nor false. But if we want to preserve the Equation--and Adams' logic--we need to avoid the Triviality Result too. In such a way we might support the thesis according to which the probability of a conditionals is its conditional probability without necessarily deny any kind of truth conditions for conditionals.

With this intent, my work considered Mura's theory trying to show why the Theory of Hypervaluated Trievent may represent a good option. I showed how it has been able to avoid the Triviality and to incorporate Adams' logic, extending it to every trievent. Demonstrating that every trievent is *simple*, Mura provided a theory able to deal with both simple and compound conditionals, overcoming a limit of Adams's theory. Indeed, even if Adams considered compound conditionals not treatable at all--probably because they are too complicate--, we can find many examples where such sentences are unquestionably used in every day reasoning.

However, Mura's account is still a work in progress and it is not totally free from limits. I presented its problems, but also its advantages. Among them, the generalization of the Theory of Hypervaluated Trievent by invoking a corpus K of beliefs, which allows to consider every conditional not indicative or counterfactual by itself but in relation to *K*. It means we can still hope to find a unified theory able to catch both kind of conditionals.

Even Stalnaker introduced some (contextual) restriction for a set of worlds, so that there is no such a logical difference between *K* and that set--at most, the difference would concern the metaphysical level. Nevertheless, *K* should help to avoid those problems related to a "possible world". For example, problems about defining an elusive concept like that of "similarity between worlds". Although Stalnaker said he had never explicitly talked about a "similarity relation", given that this notion is certainly invoked in some way, I think it should be definitely formalized. Indeed, Lewis tried to give a properly logical definition for such a tricky idea but it did not sound exhaustive.

In conclusion, I wanted to show that it would be worth to analyze the problem of conditionals by a different point of view, hoping that my work could have helped in treating these complex sentences.

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